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THESIS

IMPROVING THE NPSCUL STRUCTURE: OPTIMIZING THE MASS

by

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September 2010

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This thesis presents the mass optimization of the Naval Postgraduate School Cube Satellite Launcher. The current structure weights 170.63 lbs and can hold up to 24 Cube Satellites. The weight of the whole system, which includes the launcher and all contained satellites, cannot exceed 177 lbs or be lighter than 169 lbs. By reducing the mass of the launcher, the freed weight can then be reallocated to the Cube Satellites. This will increase the maximum allowed weight per satellite.

The current design has six pounds of available free weight. During the course of the analysis, it is found that removal of unused portions of the four side walls of the launcher structure will reduce the weight without undermining the strength. Additionally, when looking at the use of different metal options for the launcher, it is found that the structure needs to maintain the use of an aluminum alloy. Maintaining space-grade aluminum in conjunction with the removal of unused sections of the side walls frees an additional four pounds. This totals ten pounds that can be allocated back into the Cube Satellites.

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IMPROVING THE NPSCUL STRUCTURE: OPTIMIZING THE MASS

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LIST OF ACRONYMS AND ABBREVIATIONS

ADAMSat	AS&T Technology Demonstration And Maturation Satellite
CubeSat	Cube Satellite
Cal Poly	California Polytechnic State University
COTS	Commercial off-the-shelf
FEM	Finite Element Model
MA	Modifiable area
MATLAB	Analysis software
NPS	Naval Postgraduate School
NPSCuL	Naval Postgraduate School Cube Satellite Launcher
NASA	National Aeronautics and Space Administration
P-POD	Poly Picosatellite Orbital Deployer
U	Unit of 10 cm^3 picosatellite
VMA	Volume of Modifiable Area

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I. HISTORY AND PURPOSE

A. SATELLITES

The space age kicked off in 1957 with the Russian launch of Sputnik. Sputnik-1 was immediately followed one month later with Sputnik II, which carried a dog named Laika. The United States countered with the launch of Explorer 1 in 1958, which went on to discover the Van Allen radiation belts. These feats led to the launch of Echo 1, an aluminum-coated plastic sphere that passively reflected voice and picture signals. Next was the first true communications satellite, Telstar. Telstar actively relayed communications between locations that were at great distances from one another [1]. Over the years, the use of satellites has evolved, and now they function in many different roles. Satellites are used for navigation, as relay stations for radio and television, communications, weather, observations of the Earth and space, and for wars. The use of these satellites will not diminish; rather, as time progresses, there will be new inventive things that can be done with a satellite. Additionally, as resources become scarcer, the satellite will need to get smaller and cheaper.

The Galileo, Cassini and Hubble spacecrafts had life cycle costs over one billion dollars. The Clementine space craft cost $85 million and the Lunar Prospector cost $63 million. After the Cold War ended, the exclusivity of space to a select few countries started to change. There became a commercialization of space, from the previously military/government specific one. There was an increase in

the availability for launch services and a dedicated launch vehicle was no longer required. It became cost effective to get as many different companies, or entities, on one launch vehicle as possible to offset the costs. Large satellites, such as in Figure 1, can cost from $5,000 to $20,000 per kilogram to launch, and from $30,000 to $160,000 per kilogram to build [1]. One driving force in the design cost for satellites are the use of space-qualified components.

Figure 1. JCSAT-12 being built. From [2]

Almost all companies, including the U.S. government, require the components of a space craft or satellite to contain space-qualified components. These components must meet a military or NASA (National Aeronautics and Space

Administration) standard for the material, fabrication, storage and transport. These requirements significantly increase the price of a satellite. Additionally, anywhere from two to four models must be made; one for testing, one for flight, a possible flight spare and a possible engineering model [1]. Costs need to be cut wherever possible. Decreasing the size requires more advanced technology that is smaller than its predecessors. Satellites need to be lighter and more efficient. The use of commercial off-the-shelf (COTS) products dramatically reduces these costs. Multiple spares can be purchased for a fraction of the price of designing and fabricating required circuitry. These steps have led to an increase in the use of satellites by universities for education and research [1].

Satellites are classified by weight into size categories. Cube Satellites, Figure 2, which are known as CubeSats, can be nano-satellites (weighing from 1-10 kgs) or pico-satellites (weighing less than 1 kg) and are generally 10 centimeters cubed. They are orders of magnitude smaller than their full satellite cousins. The idea of a CubeSat is not only important to the space industry, but to education as well. CubeSats are becoming important and useful to high schools, colleges, and universities. Students are designing, building and launching these small objects through their own programs. The knowledge they gain in these programs prepares them for joining the industry’s satellite community. Their small size allows CubeSats to be placed as a secondary payload on an already scheduled launch vehicle, vice having the dedicated launch required for larger satellites. For the industry, a CubeSat can be launched on a budget of only $100,000 and the launch can take place years before a full-

sized satellite could [3]. For student programs, there is a discount to stimulate research and development for new innovations, where a CubeSat can be launched for as little as $40,000 [4]. This price allows new and radical technologies to be tested in space that would not normally take place due to the large monetary investment that the industry has on a given launch. Industry needs systems that are guaranteed to work and do not have the money to spend on a “what if.” These CubeSats allow for a much lower cost of space testing for these advancements. Launch opportunities still remain an issue. Two schools, California Polytechnic State University (Cal Poly) in San Luis Obispo, CA, and the Naval Postgraduate School (NPS) in Monterey, CA, have been instrumental in assisting in this area.

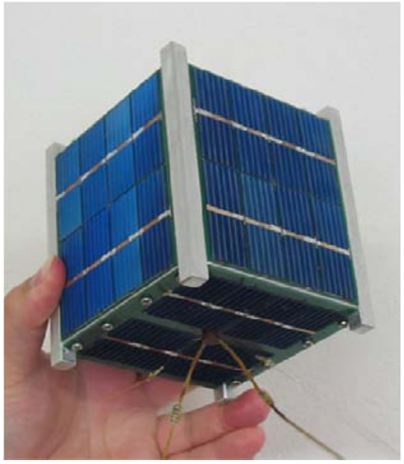

Figure 2. Size of a Cube Satellite. From [5]

B. STUDENT INVOLVEMENT

Cal Poly has standardized the industry by creating the Poly Picosatellite Orbital Deployer, or, as it is commonly known, the P-POD. The P-POD (Figure 3) can hold up to three

units (3-Us) of CubeSats. A single unit of CubeSat for the P-POD is defined as a 10 cm x 10 cm x 10 cm satellite. As such, a two unit (2-U) satellite would be 10 cm x 10 cm x 20 cm. Each P-POD contains and shelters the satellite(s) throughout the launch process and once orbital altitude is reached, deploys the cargo. The P-POD system is self contained and standardized. CubeSats going into the P-PODs have maximum weight limits and center of gravity constraints. Once CubeSats are built, they are sent to Cal Poly, where they are incorporated into the P-POD and sent to the launch site for transportation into space via a previously arranged launch vehicle [6].

Figure 3. Cal Poly Mk II P-POD. From [7]

A common problem in satellite launches is missed launch opportunities. Parts not being delivered on time, lack of adequate access to clean rooms, failed inspections/tests, and countless other reasons also can cause the delivery date of satellites to be missed. Current rocket launch procedure, the method of CubeSat delivery to space, requires a hard and fast deadline for delivery of all cargo. This deadline is

essential to calculations of fuel and launch criteria for the rocket. If a CubeSat/P-POD fails to be delivered on time, weight equivalence is used and it is no longer possible to get onto that flight. The flight opportunity is missed and must be rescheduled. Flight opportunities are rare and that is why the P-POD is crucial. This allows three CubeSats to be launched instead of one.

C. NPSCUL

Currently, there are about 40 to 50 university programs developing CubeSats [4]. All of these programs would like to test out their systems in space but, even with Cal Poly’s three-unit launcher, there are still not enough opportunities. As such, NPS has stepped in to assist with the creation of the NPS CubeSat Launcher, or NPSCuL as seen in Figure 4. NPSCuL, in short, is a unit that contains eight P-PODs, increasing a rocket’s delivery potential from 3-Us of CubeSats to 24-Us. Therefore, when one launch opportunity comes along, multiple educational programs can get on board. This can also lower the individual cost per launch.

Figure 4. Image of NPSCuL.

NPSCuL has already been designed, built and qualification tested to ULA ABC launch environment [8]. NPSCuL was designed specifically for ESPA (EELV or Evolved Expendable Launch Vehicle secondary payload adaptor). The ESPA ring (Figure 5), allows for six payload attachments. NPSCuL can be connected to ESPA compatible fixtures on the Atlas, Athena, Delta, Falcon 9, Falcon 1e, Minotaur IV, Taurus and European Space Agency’s VEGA rockets [9]. On the Atlas V, one possible union of the launcher and rocket is through the United Launch Alliance (ULA) Aft Bulkhead Carrier (ABC) [10] (Figure 6). The NPSCuL structure consists of a 5-sided box, 8 P-Pods, a sequencer, and fasteners. All specifications for the current design of NPSCuL, as tested, are shown in Table 1.

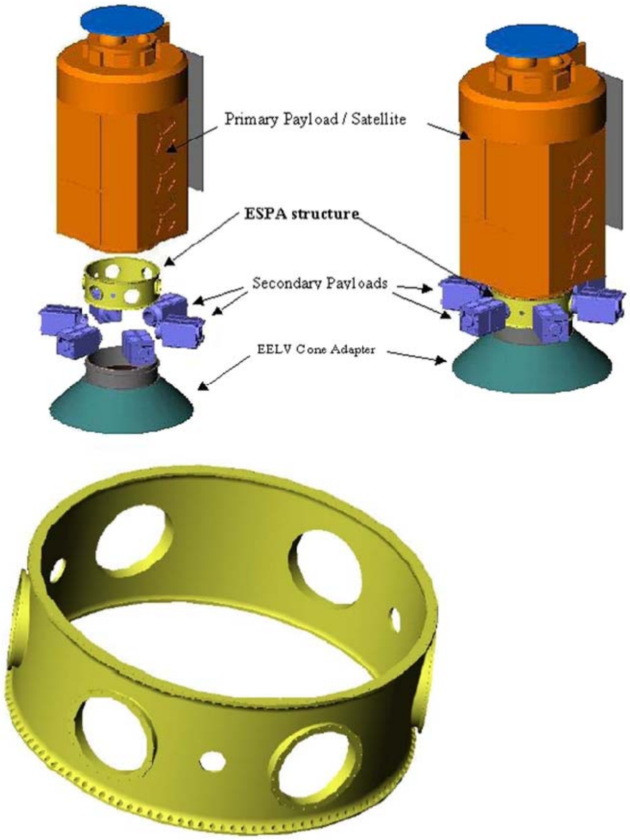

Figure 5. ESPA ring on the EELV. From [13]

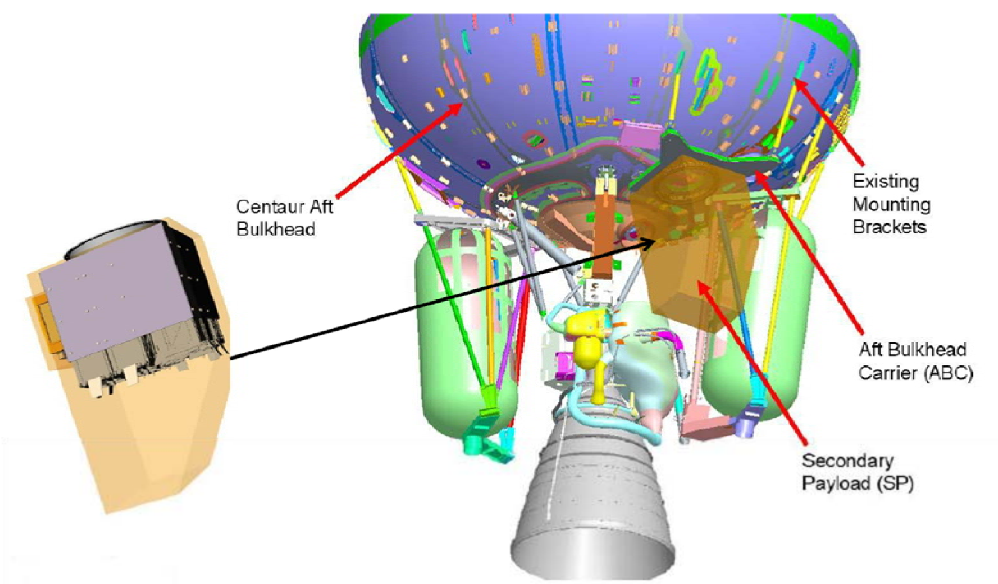

Figure 6. ULA ABC. From [14]

Table 1. Current NPSCuL structure data. From [11] and [12]

NPSCuL	**lbs**
8 P-Pods	116.84
Sequencer plus fasteners	8.80
Box plus fasteners	42.15
Harness	2.90
Total weight of NPSCuL	170.63
Max allowable weight of structure	177.00
Current usable weight	6.37

Even though there is a current design for the NPSCuL, it could use some improvement. While technology is getting smaller and better, it also allows satellite designers to put more equipment into the CubeSat structure. With the advancement in technology making parts smaller, developers want to fit more into the CubeSat. As such, this increases the demand of the previously allotted 1.33-kg weight allowance [6]. By making the structure lighter, this will allow the freed weight to be applied back into the CubeSats. The design constraints are shown in Table 2. The weight constraint for launch still remains with a maximum weight of NPSCuL, including the CubeSats, not exceeding 177 lbs or being lighter than 169 lbs [11].

Table 2. Design constraints for NPSCuL system. From [11] and [15]

Max weight of structure	177.00	lbs
Minimum weight of structure	169.00	lbs
Weight of a side of the box	7.44	lbs
Expected g-forces in X direction [12]	5	g's
Expected g-forces in Y direction [12]	5	g's
Expected g-forces in Z direction [12]	7	g's
Density of Al 7075-T7351 [11]	0.101	lbs/in^3
Modulus of elasticity [11]	10.3E6	psi
Yield Stress [11]	57	ksi

II. THE DYNAMIC STRUCTURE

A. LAUNCH EFFECTS

NPSCuL must get to space to be effective. The current method is to go via rocket and launching on a rocket is not a smooth ride. There are strong forces, up to seven times the gravitational constant, and random vibrations. The natural frequencies of the structure are of the utmost interest. At these natural frequencies, the CubeSat launcher can see a large amplification of forces, which in turn will be transmitted to the CubeSats. Even though the launcher structure can easily withstand these forces, the safety of the CubeSats inside the launcher becomes a question. The satellites, though built tough for the space environment, still have sensitive sensors that can be damaged. The current design for NPSCuL has already been analyzed using a Finite Element Model (FEM) in NX I-DEAS 6; a modeling software. The first six modes of the structure are selected for reference in this research.

B. MODAL SHAPES

The modal shapes refer to the movement of the structure at the system's specific natural frequencies. Of NPSCuL's lowest six modes, this analysis will only show modes 1, 2, 3 and 6. Mode 1 is the lowest natural frequency, and modes 2, 3, and 6 have significant center of gravity movement and structural displacements. Figures 7 through 10 show the analytical display of the analysis of these frequencies [17].

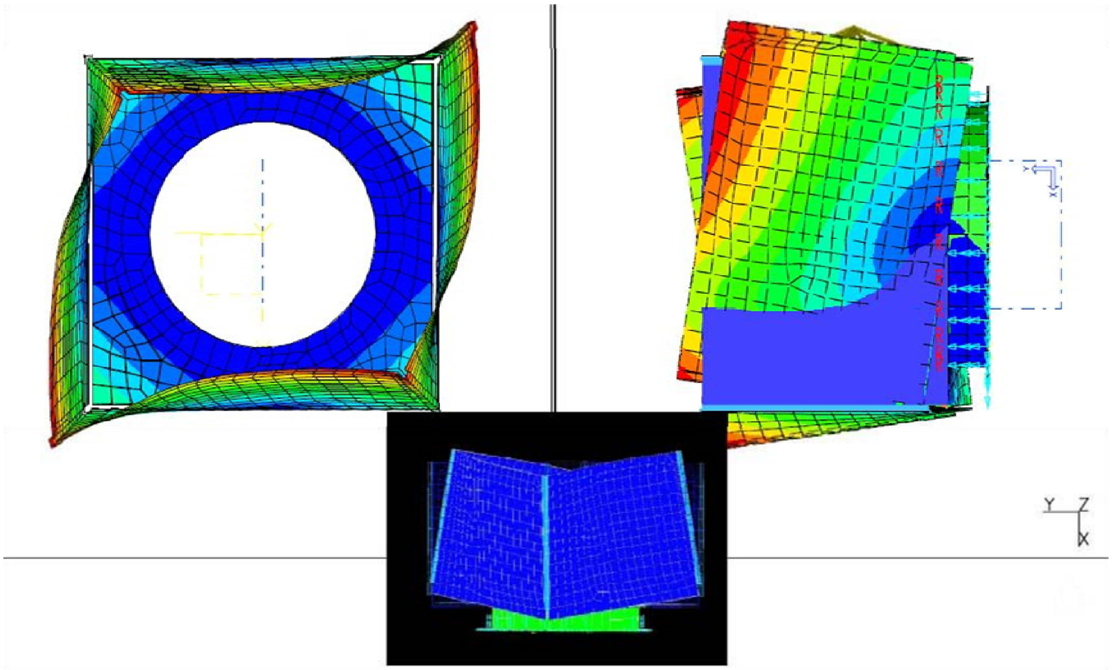

Figure 7. Mode 1 analyses. From [17]

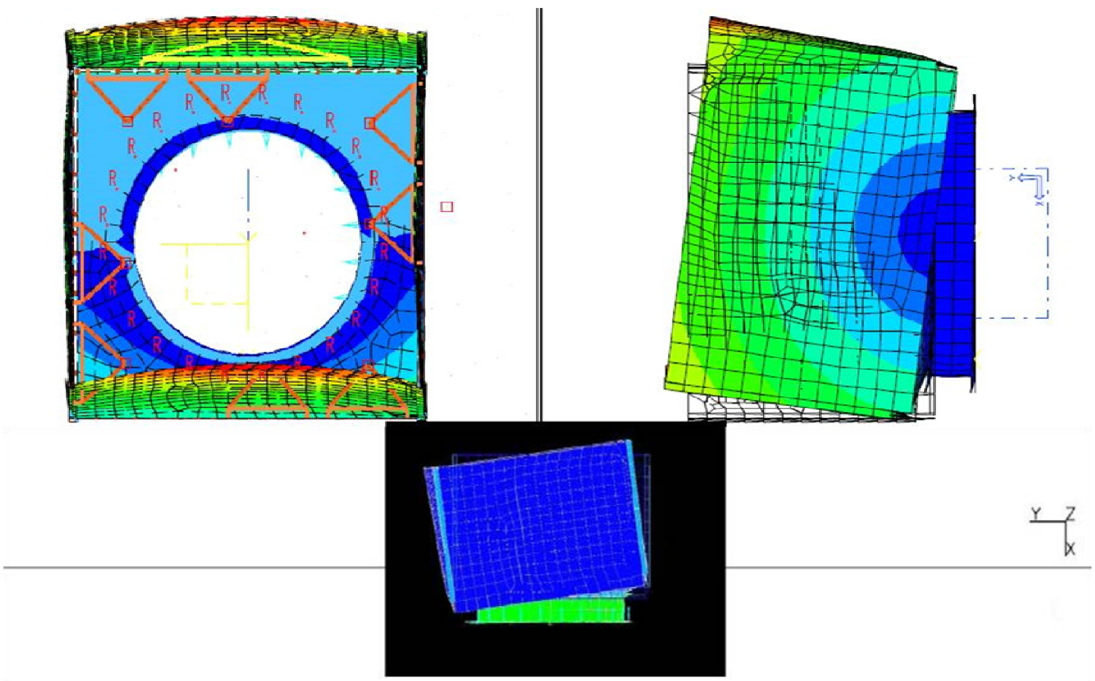

Figure 8. Mode 2 analyses. From [17]

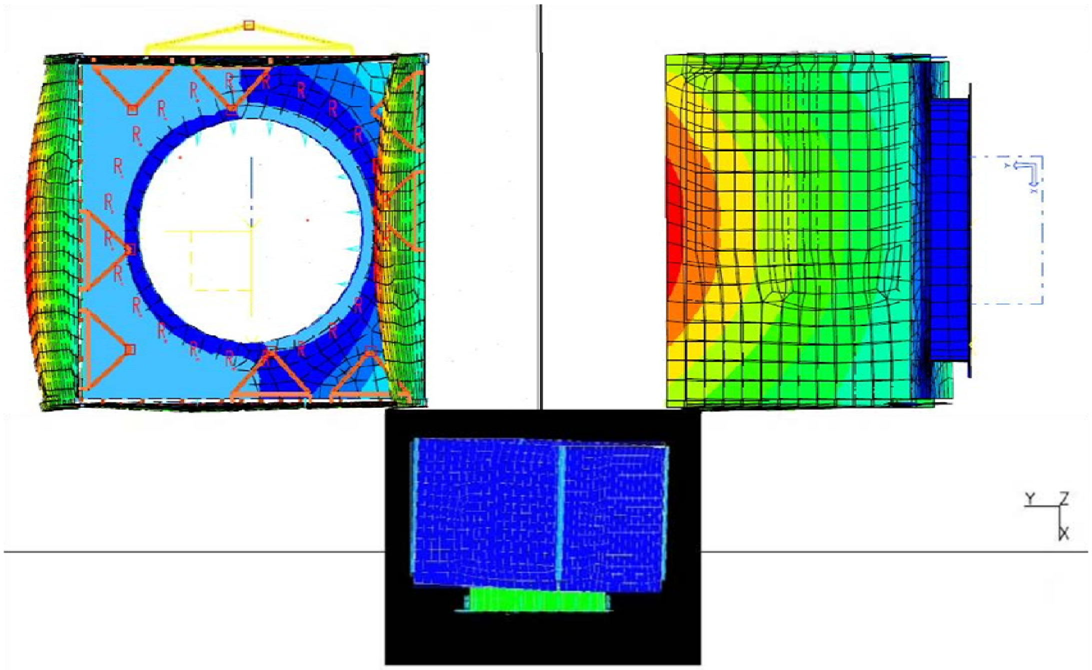

Figure 9. Mode 3 analyses. From [17]

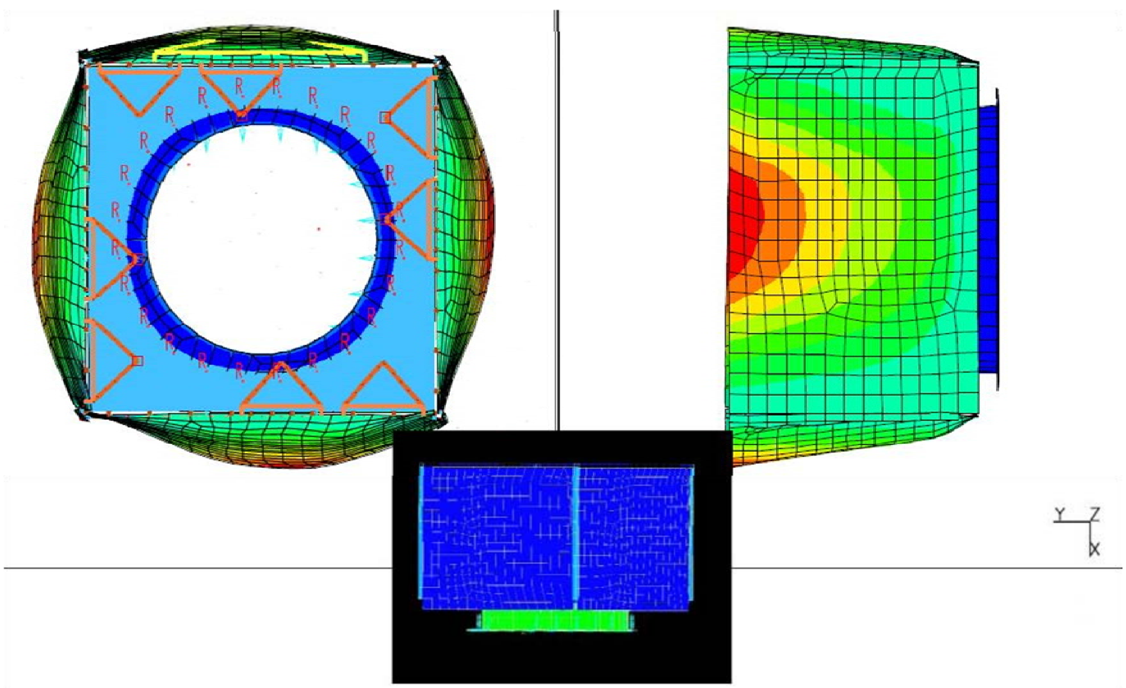

Figure 10. Mode 6 analyses. From [17]

Mode 1 is considered “corner flexing” [17] and is the lowest natural frequency for the structure. The requirements for the structure is to have a first natural frequency no lower than 35 Hz [15]. The first natural frequency was analytically evaluated at 41.39 Hz. Figure 7 shows the rendering of movement for mode 1. Mode 2 and mode 3, 53.75 and 54.73 Hz, respectively, are modes that are a “rocking motion” in the X-Y plane, as seen in Figure 8 and the Y-Z plane, as seen in Figure 9. Mode 6 is a “po-go” motion of the entire structure up and down onto the adaptor ring. This is shown in Figure 10 [17].

C. VIBRATIONS TESTING

The next step is to take the results determined by analytical methods and verify them with a vibrations test (Table 3). The engineering model of NPSCuL was loaded with mass models for the P-PODS and, with the assistance of tri-axial accelerometers, the actual natural frequencies were found. Table 4 shows the results of this test [10] and Table 5 shows this data compared to what was expected from the analytical results. Figures 11-14 show the structural movement of the first and primary modes (modes 1, 2, 3 and 6).

Table 3. Comparison of I-DEAS modeling and test results.

Mode	Analytical Frequency	Tested Frequency	Percent Error
1	41.39	42.96	3.65
2	53.75	57.83	7.06
3	54.73	59.32	7.74
4	84.68	83.93	0.89
5	87.62	85.25	2.78
6	95.06	100.5	5.41

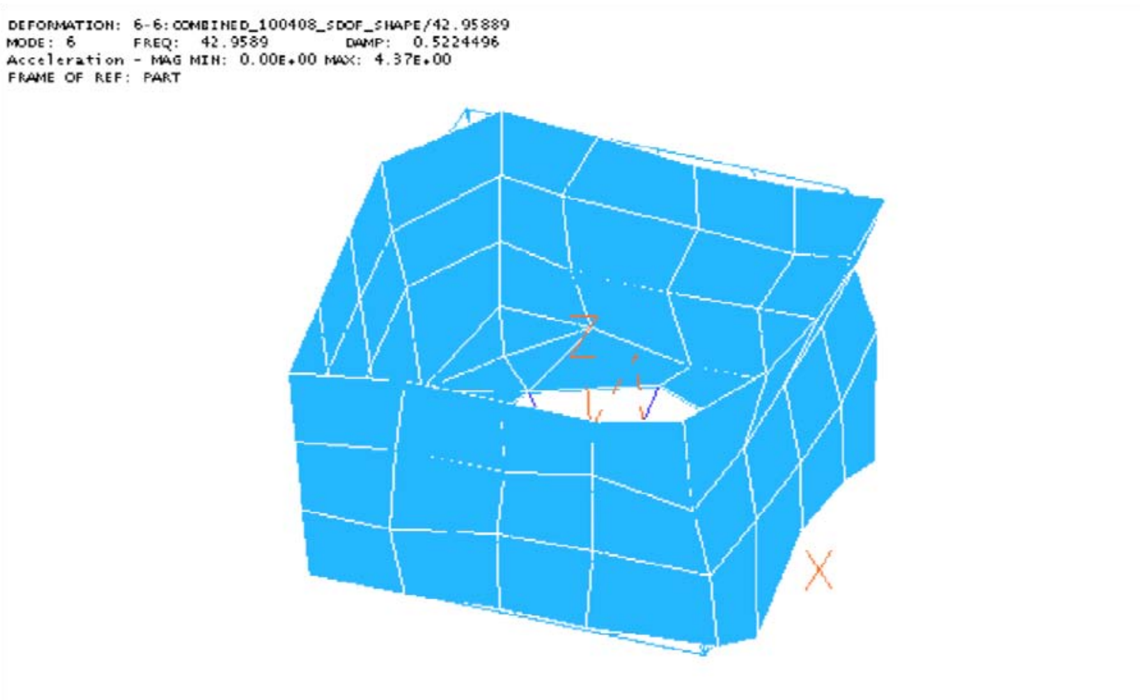

Figure 11. First natural frequency, 42.96 Hz, structural deflection. From [10]

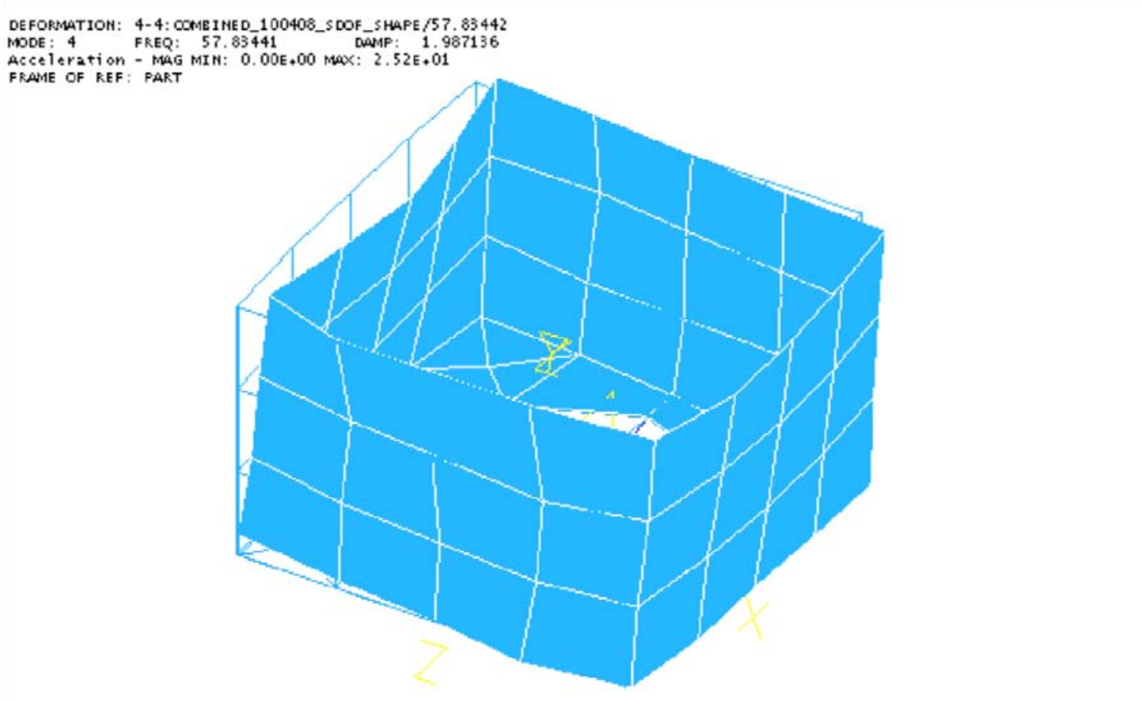

Figure 12. “Rocking” natural frequency, 57.83 Hz, structural deflection. From [10]

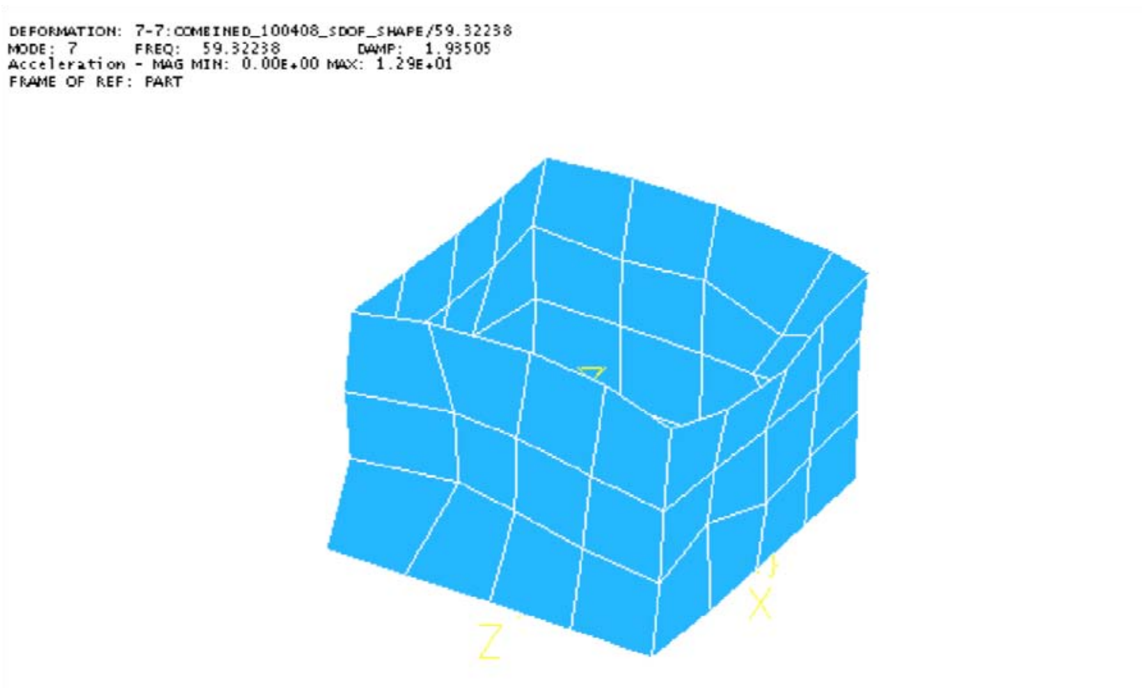

Figure 13. “Rocking” natural frequency, 59.32 Hz, structural deflection. From [10]

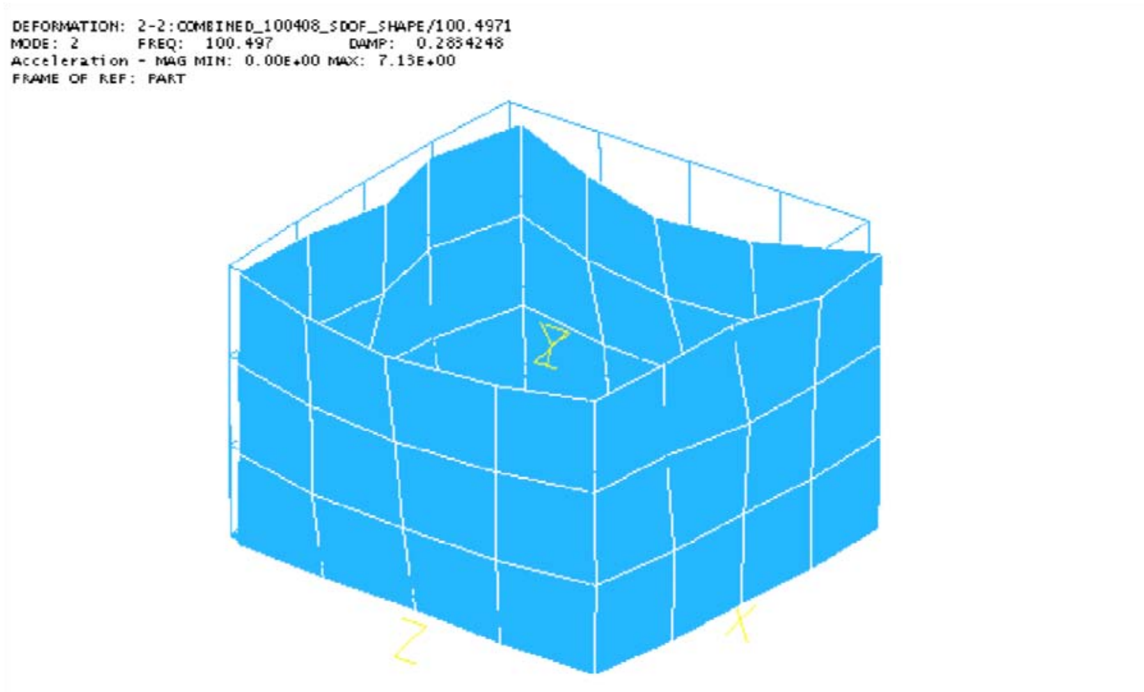

Figure 14. "Po-go" natural frequency, 100.50 Hz, structural deflection. From [10]

D. OBJECTIVE

The goal is to reduce the mass of the current structure. The NPSCuL is designed to create a way to launch more CubeSats per launch opportunity. The design shape was determined by fitting the structure into the given volume and mass constraints. The following analysis is the first step in optimizing the mass of the structure to see where weight can be removed without unduly reducing its strength. Once a reduced weight is achieved, further analysis to include dynamic response can be made with the new design.

There are two possible directions for weight removal: modify the current structure, or completely redesign it. The first option of modifying the structure is the cheapest and fastest way as the time, materials, and manpower do not have

to be spent to start over. An analysis needs to be done on the weight of the system to see if weight can be removed. This will be the initial iteration at looking at modifications to the structure. Once weight can be removed, it can either be integrated back into the CubeSats’ weight allowance or remain off. Additional alternatives can include external stiffeners, a damping system, or modifications to the adapter ring. All of these options eventually go toward reducing the total mass of the launcher or, in the case of the alternatives, will reduce the forces seen in the CubeSats during launch.

The second option would be to change the structure from its current box configuration and to start with an entirely new design. Using a different shape or reducing the number of CubeSats the structure can hold are all possible alternatives, but are beyond the scope of this analysis.

III. SIMPLIFICATION OF THE STRUCTURE

A. SINGLE WALL

The first simplification needed is to take the four-walled box and look at one single wall. As seen in the Figure 15, a single wall has two P-PODS attached onto the plate on the right-hand side. This area is not going to be modified since nearly a quarter of the weight of the structure is concentrated in this region and there is a large presence of fasteners. The dots represent the locations of the fasteners for the P-PODS, and the area to be modified is the 6.365-inch by 12.0-inch section on the left. For this analysis, the wall depth remains at a uniform 0.25 inches throughout the wall.

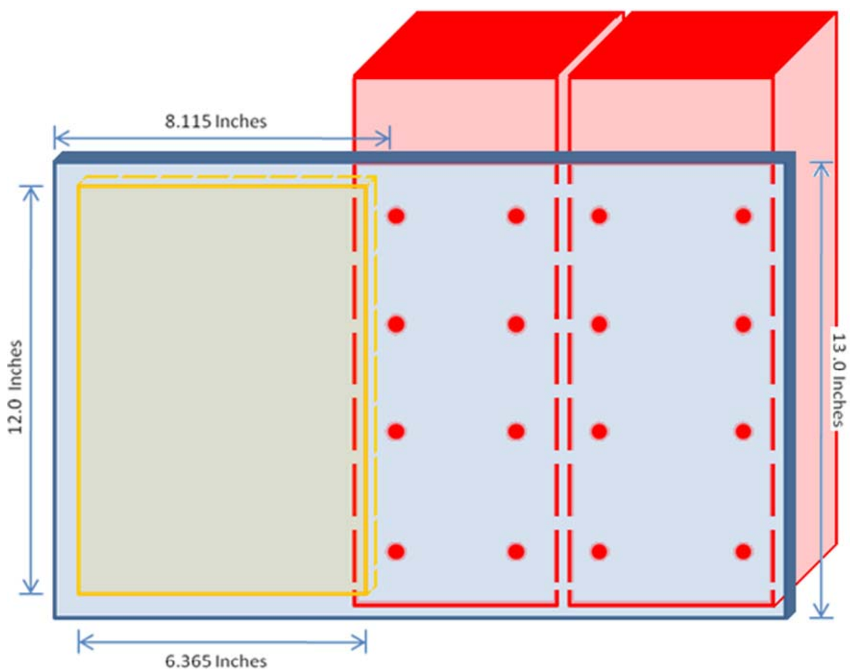

Figure 15. Two-dimensional view of side wall, including the modifiable area (MA).

B. COMPARISON TECHNIQUE

Modifying the wall volume of the current structure, while maintaining all remaining dimensions, allows an analytical comparison to the original test data. The original structure, of a solid Al 7075 wall plate with dimensions of 18 inches by 13 inches by 0.25 inches, is used as the baseline when comparing the total mass of a full wall. When comparing solutions for the modifiable area (MA), the baseline is an Al 7075 wall plate with dimensions of 6.365 inches by 12 inches by 0.25 inches. Analyzing the desired alternatives would show an increase or decrease in mass and/or an increase or decrease in stiffness. The following analysis determines the baseline of the MA by using Al 7075.

C. FLAT PLATE ANALYSIS

To analyze the stress and deflection seen in the MA, the gravitational forces must be turned into a distributed load across the top of the wall. Once the equivalent force is calculated, the stress and deflection can be determined and from there the stiffness can be calculated. The maximum weight of the structure is 177 lbs. During launch, the structure can see up to 5 gs of force in the X-direction, 5 gs of force in the Y-direction and 7 gs of force in the Z-direction. Using the static-equivalent to these dynamic forces, the following calculations show why 1200 lbs is used. The 6.365 inch by 12 inch MA is approximately one-third of the total volume of one side wall. The factor of safety used for this calculation is two.

$$1 \text{ gravitational unit} = 9.8067 \text{ N/kg} = 1.0000 \text{ lb}_f/\text{lb}_m$$

$$\text{Combined load} = \sqrt{(5^2+5^2+7^2)} = 9.95 \text{ g's} = 9.95 \text{ lb}_f/\text{lb}_m$$

$$9.95 \text{ lb}_f/\text{lb}_m * 177 \text{ lb}_m = 1761.15 \text{ lb}_f$$

$$1761.15 \text{ lb}_f * (1/3) = 587.05 \text{ lb}_f$$

$$587.05 \text{ lb}_f * 2 \cong 1200 \text{ lb}_f$$

Now, the full unmodified wall can be analyzed. The current structure is made from Aluminum 7075-T7351 and a full plate analysis yields the baseline data for comparison to volume removal in the modifiable area. The material properties are found in Table 4.

Table 4. Aluminum 7075-T7351. From [18]

Modulus of Elasticity (E)	10.3×10^6	psi
Density	0.101	lb_m/in^3
Yield stress (σ_Y)	57	ksi
Ultimate stress (σ_U)	68	ksi

The mass of a single full wall is equal to

$\rho*t*l*h = 0.101 \text{ lb}_m/\text{in}^3 * 0.25 \text{ in} * 18 \text{ in} * 13 \text{ in} = 5.909 \text{ lb}_m$

The mass of the modifiable section of a single wall equals

$$\rho * t * l * h = 0.101\ lb_m/in^3 * 0.25\ in * 6.365\ in * 12\ in$$
$$= 1.929\ lb_m$$

Maximum stress is [16]

$$\sigma = F/A = 1200\ lb_f\ /\ (0.25\ in * 6.365\ in) = 754.12\ psi$$

The stiffness in the area to be modified [16]

$$k = [AE]/l = [(0.25\ in * 6.365\ in) * 10.3x10^6\ psi]\ /\ 12\ in =$$
$$1366x10^3\ lb_f/in$$

These values will be essential in further calculations and show that the modifiable area is approximately one-third of the total mass of one side of a wall. It is also important to keep in mind that one of the four side walls contains the sequencer and will not be modified. This wall is shown in Figure 16.

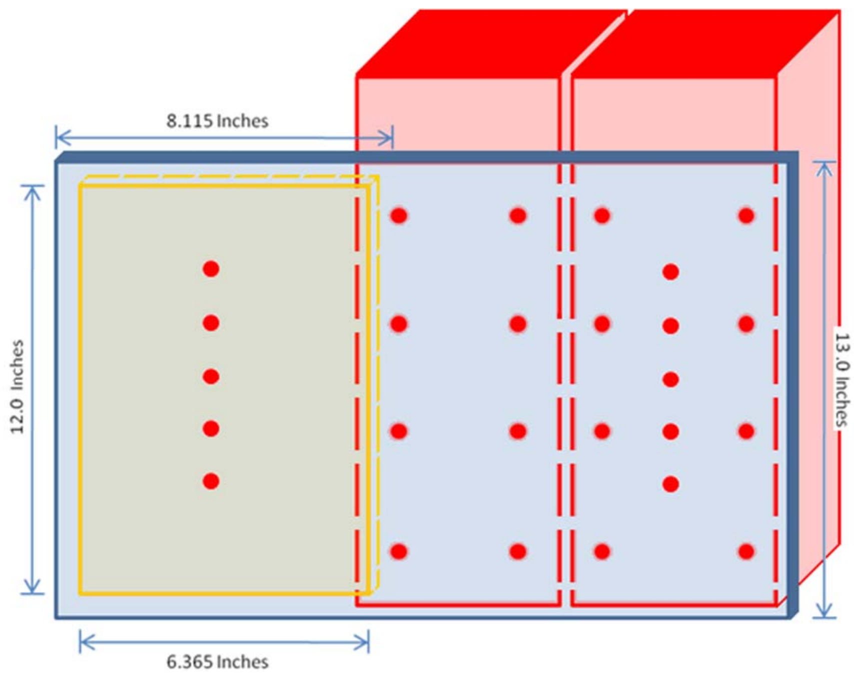

Figure 16. Two-dimensional view of side wall with sequencer bolt holes.

IV. MASS REDUCTION OF THE ALUMINUM WALL

A. REMOVAL OF WALL VOLUME

Due to the presence of corner flexing in the current structure during modal movement [10] and [17], simply removing the 6.365" by 12" section is unwise. The complete removal would further undermine the corner regions, adding to the flexing already seen. There need to be diagonal stiffeners in this region. The stiffness of this modifiable area started as 1366×10^3 lb_f/in; however, once the center section is removed, it becomes approximately

$$k = [AE]/l = [(0.25 \text{ in} * 1 \text{ in}) * 10.3 \times 10^6 \text{ psi}] / 12 \text{ in} = 214.6 \times 10^3 \, lb_f/in$$

This reduces the original stiffness by nearly 85%. Table 5 shows the comparison of a full wall of Al-7075 compared to removing the entire modifiable area. Figure 17 shows this data graphically.

Table 5. Comparison of removing all of the modifiable area.

	Mass	Percent of mass	Stiffness	Percent of stiffness
Full Wall	5.909	100	1366	100
Removal of MA	3.980	67.4	214.5	15.7

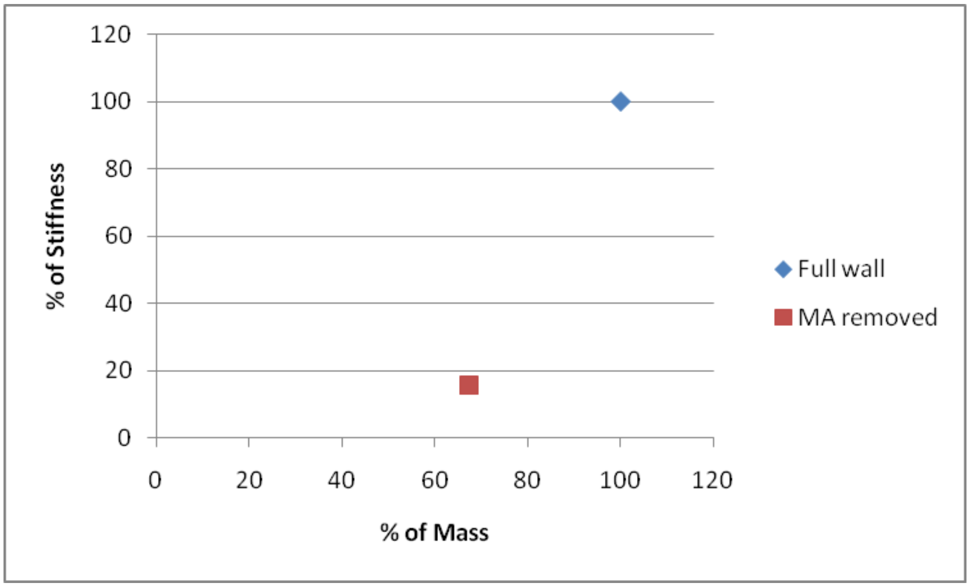

Figure 17. Visual display of full wall versus removing the MA.

This drop in stiffness is dramatic. The verification to ensure that this stiffness drop does not undermine the system is to check for buckling of the thinnest members. Checking for buckling along with verifying that the yield stress of the material is not violated and the deflections are minimal determines the suitability of the alternative. The sides of the MA are 0.5 inches wide; the wall has a thickness of 0.25 inches, and the member seeing the largest force is 12 inches long. Using the buckling criteria [19] for a beam, the critical load that can be carried before failure is calculated. This value can then be compared to the maximum force seen in the member. The results of this analysis, as shown in Table 6, explain that the removal of the entire MA results in failure.

$$P_{cr} = [\pi^2 * E * I]/L^2$$

$$\text{Where } I = [w * t^3]/12$$

Table 6. Buckling failure of full MA removal.

MA	F (lbs)	P_{cr} (lbs)	Status
0.5	600	459.6	Failure

B. TRUSS ANALYSIS

The possible solution to optimizing the mass is for material from the MA to be removed while keeping portions as support members. To do this, a Finite Element Methods (FEM) technique is utilized. FEM is a way to take a complicated problem and simplify it by putting it into a computer program to be analyzed. Three-dimensional problems can be simplified as two-dimensional ones with the use of beam elements. Using the basic MATLAB code found in [20] for a truss system, the modifiable area was turned into a truss problem. FEAPLYC2.m, FEASMBL1.m, FEELDOF.m and FETRUSS2.m were all used as the basic code and then a function file was created to run these programs with various truss designs. Only the code in the function file was modified for each variation in the design. The analysis was conducted on eight different truss designs, as shown in Figures 18-20.

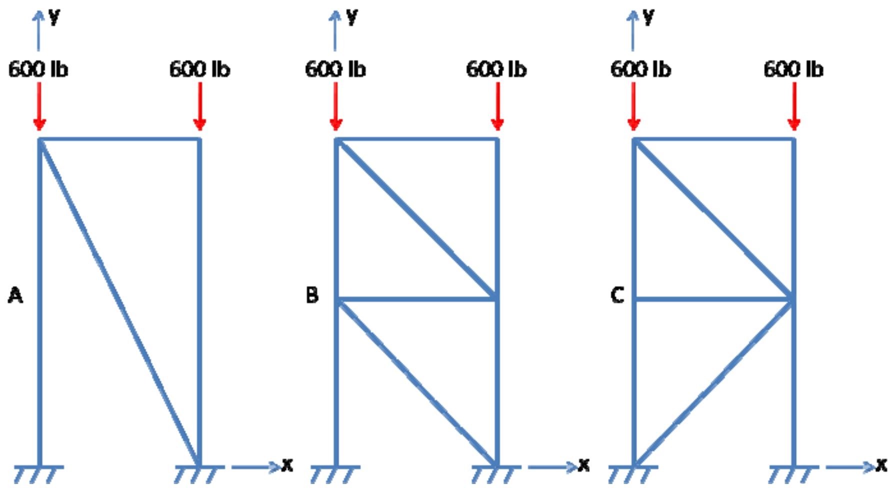

Figure 18. Truss variations A, B and C.

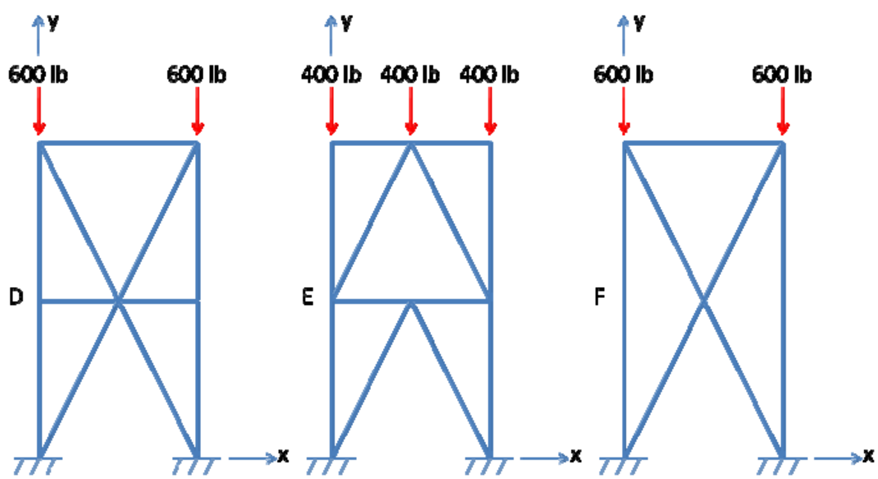

Figure 19. Truss variations D, E, and F.

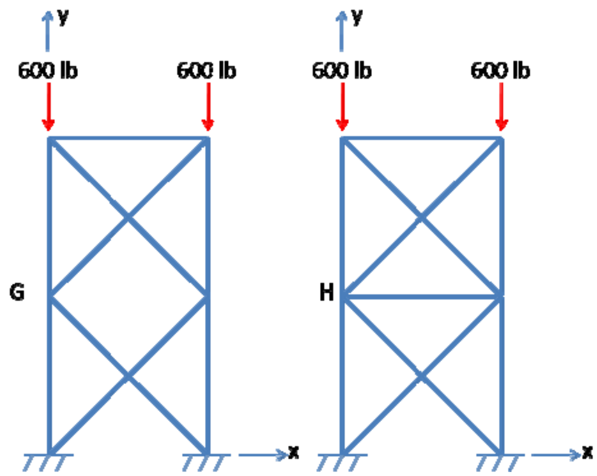

Figure 20. Truss variations G and H.

C. CALCULATIONS

Not only are all eight configurations analyzed, but additionally each configuration is evaluated using three different truss beam widths; 1 inch, 0.5 inches and 0.25 inches. For all cases, each beam remained at 0.25 inches of thickness and the beam width refers to the width of the face. An example of how the three different beam widths would look on configuration A is shown in Figure 21. The function codes for each variation can be found in Appendix A. Each file is designated first by the letter assigned to the configuration as shown in Figures 18-20, and secondly by the size of the beam width. FEMA1 denotes the FEM analysis of a 1-inch beam width, FEMA5 denotes the 0.5-inch beam width and FEMA25 denotes the 0.25-inch beam width.

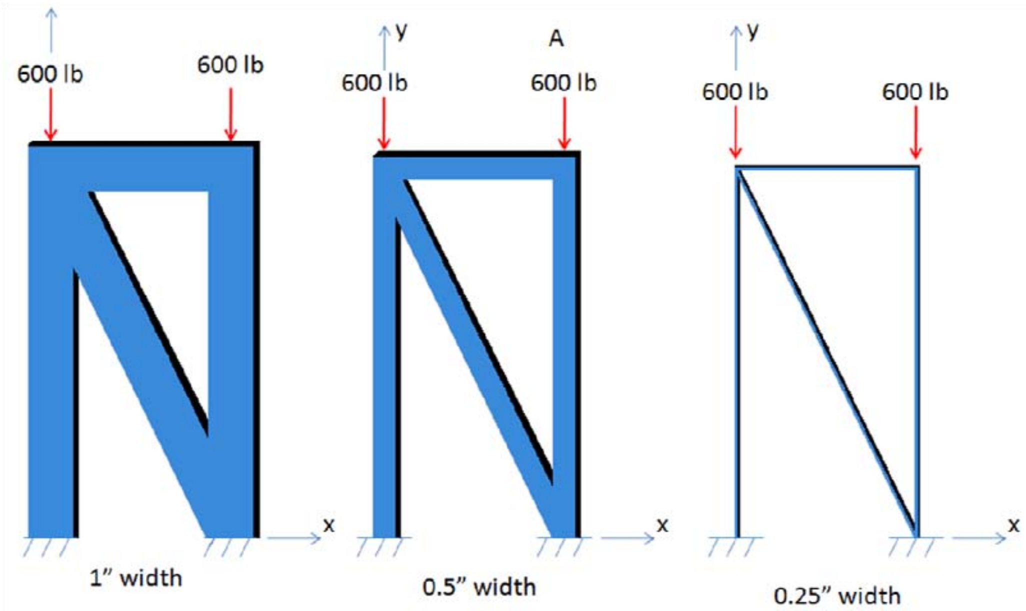

Figure 21. Example of what different beam widths look like in the MA.

The output created by MATLAB contains the maximum stress in each beam element and also each node's deflections in the X and Y directions. Elements and nodes are numbered in the clockwise direction. Each node has two degrees of freedom, the X and Y directions. As such, the first node would have the designation of the X direction's deflection denoted as one, its Y direction's deflection denoted as two and node two would have X and Y directions listed as three and four, respectively. These results for all 24 permutations are found in Appendix B. From the stress, deflection, and volume of removed material, each variant's percent of total wall mass and stiffness can be calculated. The following data is used to create the output found in Table 7.

$$\sigma = F/A$$

$$k = F/\delta$$

Cross-sectional area of 1" width beam = 0.25 in^2

Cross-sectional area of 0.5" width beam = 0.125 in^2

Cross-sectional area of 0.25" width beam = 0.0625 in^2

Volume of Modifiable Area (VMA) = 0.25 * 6.365 * 12

= 19.095 in^3

Volume of Voids (VV) = Volume of material no longer present in MA

Table 7 shows the calculations for the stiffness and mass loss for each situation. The percent decrease in mass is plotted against the percent of stiffness from the original solid wall baseline. Figures 22-24 show the results for beam width thicknesses of 0.25, 0.5 and 1 inch, respectively. The most advantageous system would have the largest percent of mass reduction while having the largest percent of stiffness. As an example, in Figure 22, configuration A has the highest reduction in mass but the lowest amount of stiffness. Conversely, configuration C has the largest stiffness but is only average in mass reduction.

Table 7. Percent reduction in mass and stiffness calculations.

	Beam Width [W] (in)	δ (in)	σ (ksi)	σ (psi)	A (in^2)	F=σ*A (lbs)	k=F/δ (lbs/in)	% Stiffness of full wall	W (in^3)	% redux of MA	Mass of full wall (lbs)	% Reduction of Mass of full wall	Mass freed whole structure (lbs)
A - 1"	1	-0.0053	2.4000	2400.0	0.25	600.0	113,208	23.99	12.771	66.88	4.592	22.29	3.952
A - 0.5"	0.5	-0.0105	4.8000	4800.0	0.125	600.0	57,143	19.88	15.792	82.70	4.280	27.57	4.887
A - 0.25"	0.25	-0.0211	9.6000	9600.0	0.0625	600.0	28,436	17.78	17.408	91.17	4.113	30.39	5.387
B - 1"	1	-0.0028	2.4000	2400.0	0.25	600.0	214,286	31.39	10.946	57.33	4.780	19.11	3.387
B - 0.5"	0.5	-0.0056	4.8000	4800.0	0.125	600.0	107,143	23.54	14.739	77.19	4.389	25.73	4.561
B - 0.25"	0.25	-0.0112	9.6000	9600.0	0.0625	600.0	53,571	19.62	16.847	88.23	4.171	29.41	5.213
C - 1"	1	-0.0014	2.4000	2400.0	0.25	600.0	428,571	47.07	10.946	57.33	4.780	19.11	3.387
C - 0.5"	0.5	-0.0028	4.8000	4800.0	0.125	600.0	214,286	31.39	14.739	77.19	4.389	25.73	4.561
C - 0.25"	0.25	-0.0056	9.6000	9600.0	0.0625	600.0	107,143	23.54	16.847	88.23	4.171	29.41	5.213
D - 1"	1	-0.0017	1.4495	1449.5	0.25	362.4	213,162	31.30	8.776	45.96	5.004	15.32	2.716
D - 0.5"	0.5	-0.0034	2.8990	2899.0	0.125	362.4	106,581	23.50	13.435	70.36	4.523	23.45	4.158
D - 0.25"	0.25	-0.0068	5.7981	5798.1	0.0625	362.4	53,291	19.60	16.140	84.53	4.244	28.18	4.995
E - 1"	1	-0.0023	2.4000	2400.0	0.25	600.0	260,870	34.80	8.776	45.96	5.004	15.32	2.716
E - 0.5"	0.5	-0.0047	4.8000	4800.0	0.125	600.0	127,660	25.05	13.435	70.36	4.523	23.45	4.158
E - 0.25"	0.25	-0.0093	9.6000	9600.0	0.0625	600.0	64,516	20.42	16.140	84.53	4.244	28.18	4.995
F - 1"	1	-0.0017	1.4495	1449.5	0.25	362.4	213,162	31.30	11.367	59.53	4.736	19.84	3.518
F - 0.5"	0.5	-0.0034	2.8990	2899.0	0.125	362.4	106,581	23.50	15.043	78.78	4.357	26.26	4.655
F - 0.25"	0.25	-0.0068	5.7981	5798.1	0.0625	362.4	53,291	19.60	17.022	89.15	4.153	29.72	5.268
G - 1"	1	-0.0028	2.4000	2400.0	0.25	600.0	214,286	31.39	9.901	51.85	4.888	17.28	3.064
G - 0.5"	0.5	-0.0056	4.8000	4800.0	0.125	600.0	107,143	23.54	14.092	73.80	4.455	24.60	4.361
G - 0.25"	0.25	-0.0112	9.6000	9600.0	0.0625	600.0	53,571	19.62	16.492	86.37	4.208	28.79	5.103
H - 1"	1	-0.0023	1.9984	1998.4	0.25	499.6	217,217	31.60	8.730	45.72	5.008	15.24	2.702
H-0.5"	0.5	-0.0046	3.9968	3996.8	0.125	499.6	108,609	23.65	13.413	70.24	4.525	23.41	4.151
H - 0.25"	0.25	-0.0092	7.9936	7993.6	0.0625	499.6	54,304	19.68	16.129	84.47	4.245	28.16	4.991

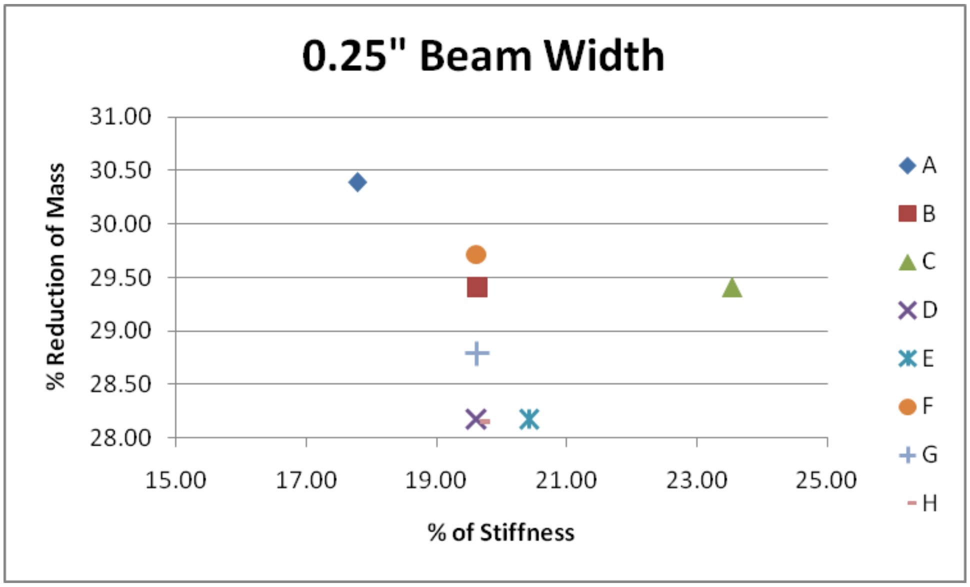

Figure 22. Reduction of stiffness versus mass for 0.25-inch beam width.

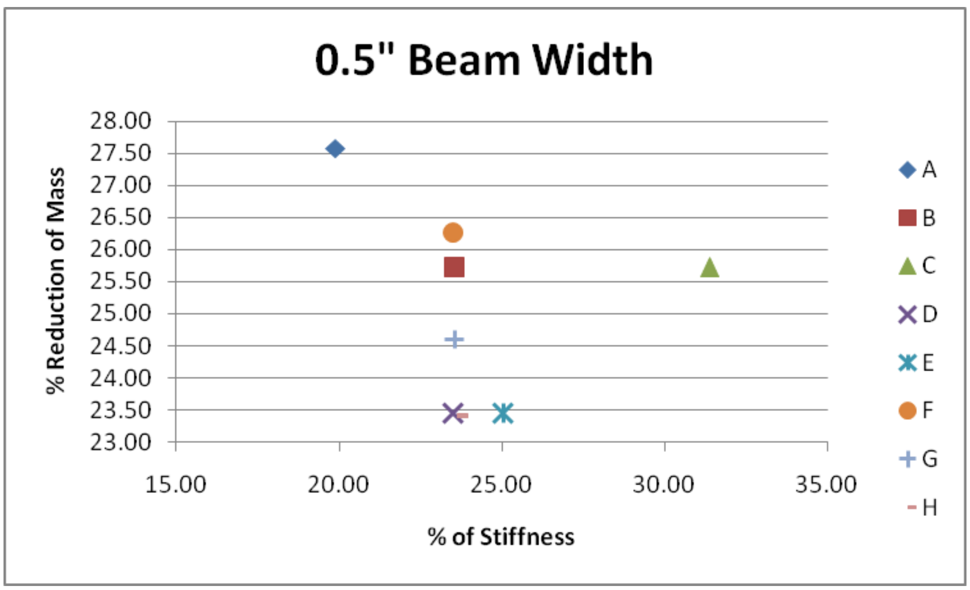

Figure 23. Reduction of stiffness versus mass for 0.5-inch beam width.

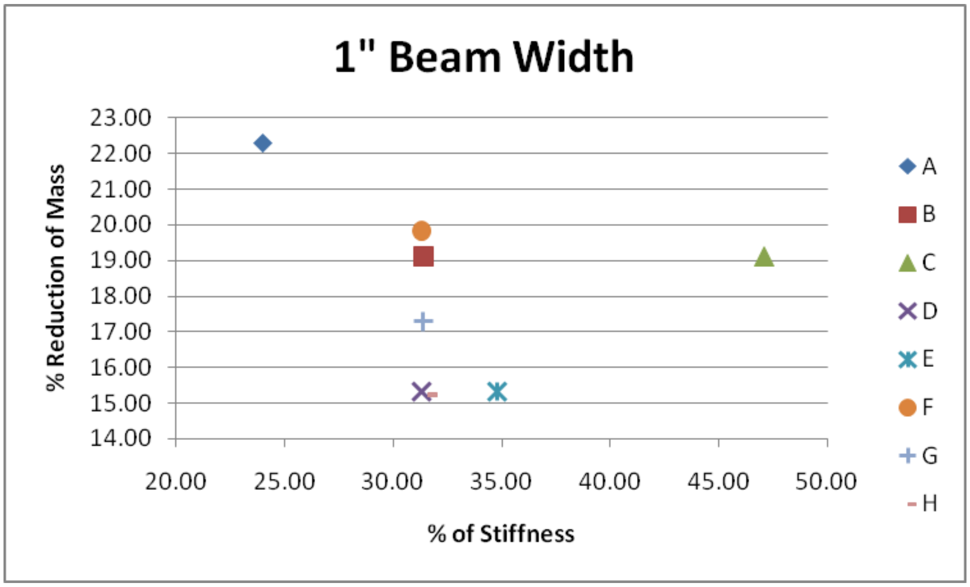

Figure 24. Reduction of stiffness versus mass for 1-inch beam width.

A definitive optimal configuration is not easily discerned. As mass is the most important parameter for this analysis, looking at each configuration's contribution toward total weight removal for the full structure is assessed. The computations in Table 7 are used to achieve Figures 25-27. The best four choices for all three beam width sizes are configurations A, B, C and F. The next step is to ensure that none of the configurations buckle in all three width sizes. This would eliminate a configuration from the analysis. Table 8 shows the calcuations and verification to whether buckling occurs. As can be seen, there are three cases where buckling would take place. Threfore, configuration A can be used only if 1-inch beam widths are used, and configuration F can be used if 0.5- or 1-inch widths are used.

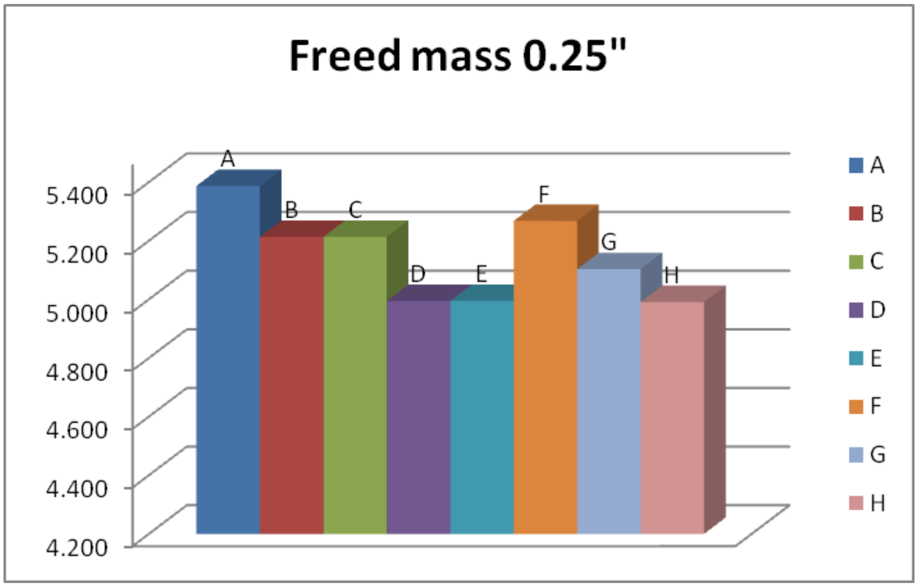

Figure 25. Total freed mass from all truss types at 0.25 inches.

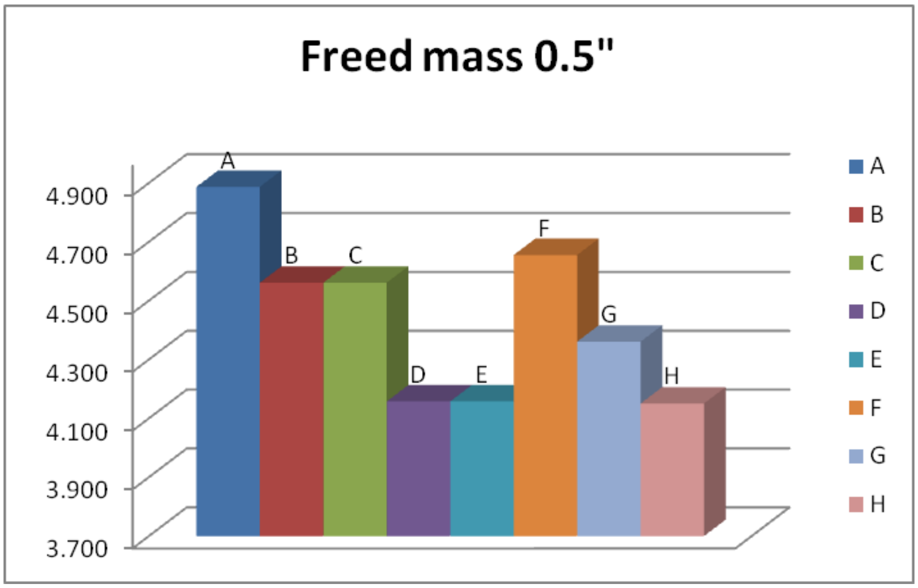

Figure 26. Total freed mass from all truss types at 0.5 inches.

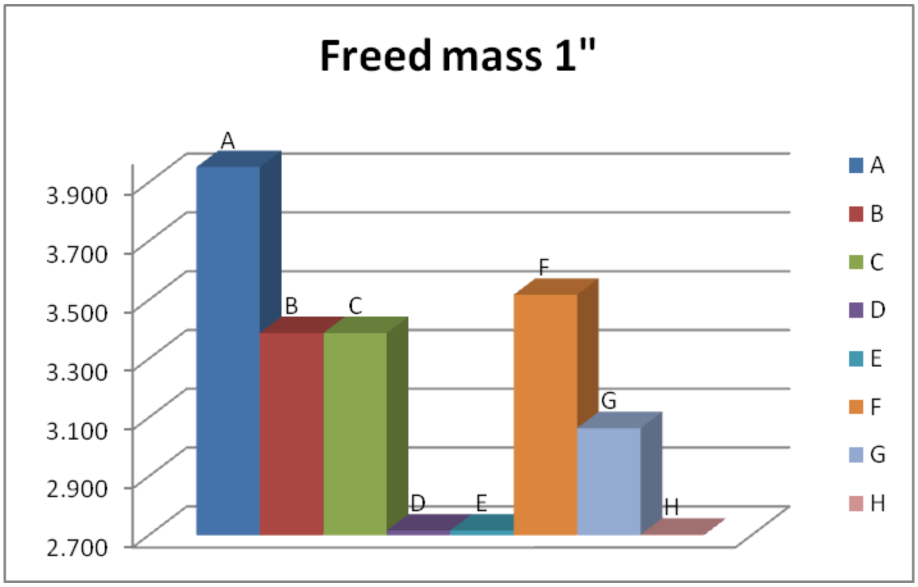

Figure 27. Total freed mass from all truss types at 1.0 inches.

Table 8. Buckling failure for each configuration.

	Beam Width - W (in)	F=s*A (lbs)	P_{Cr} (lbs)	Status
A - 1"	1	600.00	919.21	Safe
A - 0.5"	0.5	600.00	459.60	Failure
A - 0.25"	0.25	600.00	229.80	Failure
B - 1"	1	600.00	3676.83	Safe
B - 0.5"	0.5	600.00	1838.41	Safe
B - 0.25"	0.25	600.00	919.21	Safe
C - 1"	1	600.00	3676.83	Safe
C - 0.5"	0.5	600.00	1838.41	Safe
C - 0.25"	0.25	600.00	919.21	Safe
D - 1"	1	362.38	3676.83	Safe
D - 0.5"	0.5	362.38	1838.41	Safe
D - 0.25"	0.25	362.38	919.21	Safe
E - 1"	1	600.00	3676.83	Safe
E - 0.5"	0.5	600.00	1838.41	Safe
E - 0.25"	0.25	600.00	919.21	Safe
F - 1"	1	362.38	919.21	Safe
F - 0.5"	0.5	362.38	459.60	Safe
F - 0.25"	0.25	362.38	229.80	Failure
G - 1"	1	600.00	3676.83	Safe
G - 0.5"	0.5	600.00	1838.41	Safe
G - 0.25"	0.25	600.00	919.21	Safe
H - 1"	1	499.60	3676.83	Safe
H- 0.5"	0.5	499.60	1838.41	Safe
H - 0.25"	0.25	499.60	919.21	Safe

D. OPTIMAL CONFIGURATION

Now that the best configurations based on weight have been established, it is worthwhile to compare the remaining four designs. Figures 28-30 show the four configurations enlarged. Configuration A has the highest reduction of mass but the lowest stiffness. Configuration C has the highest stiffness but is tied with configuration B for the lowest

mass reduction. That leaves configurations F as a good tradeoff between mass and stiffness.

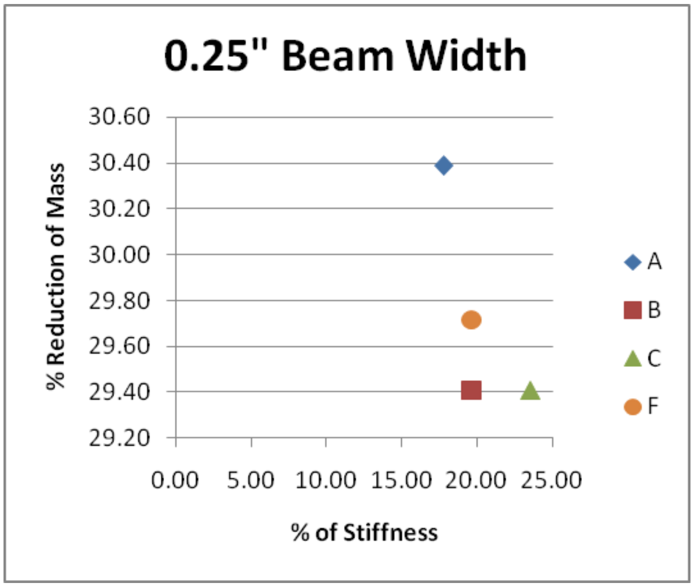

Figure 28. Comparison of four favored solutions for 0.25-inch beam width.

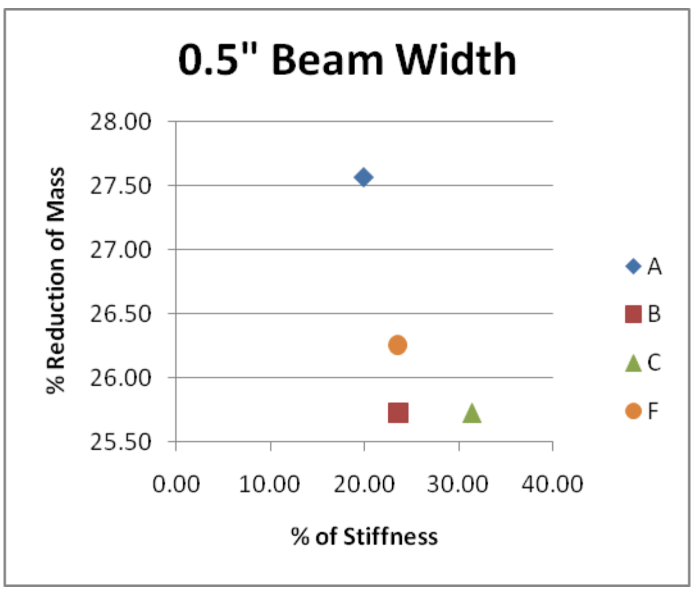

Figure 29. Comparison of four favored solutions for 0.5-inch beam width.

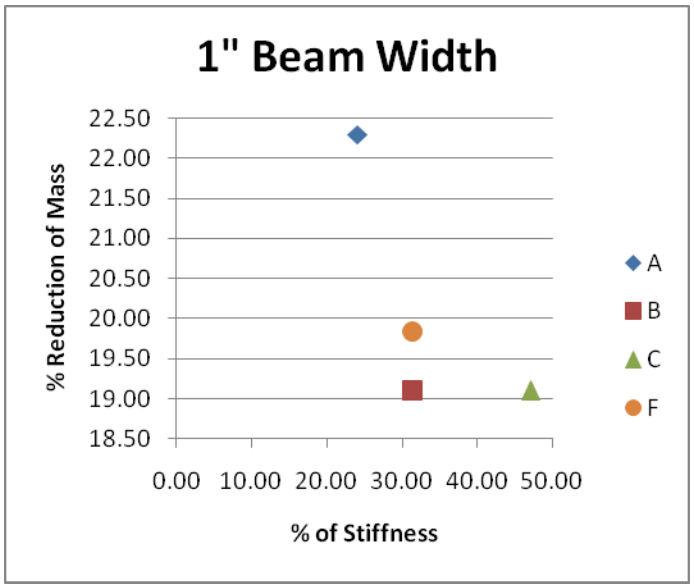

Figure 30. Comparison of four favored solutions for 1-inch beam width.

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V. CHANGE OF MATERIAL

A. CURRENT MATERIAL

The current structure is made from space-grade Aluminum, Al 7075. This metal is used due to its light weight and high resistance to stress corrosion cracking. The material is finished with a chemical conversion coating for corrosion protection, and to lower the electrical resistance [15]. A different approach to lightening the structure is to look at changing the metal. An analysis is to be conducted looking at not only lighter metals but also heavier ones as compared to Al 7075. In order to properly analyze a heavier metal, the use of the MA will be required. Lighter materials could incorporate the use of the MA but do not have to. Firstly, there are six pounds of free weight currently available for use in the NPSCuL configuration when using the space-grade Aluminum. If a heavier metal is used that exceeds this weight constraint, voids will have to be made to maintain the system within parameters. From Chapter IV, it is seen that for Aluminum, the “F” configuration is the most efficient. However, with a change in material, that configuration may change.

B. OTHER MATERIALS

Using a list of general, known and available metals [19], a simultaneous assessment can be made of different metals with different beam widths. Appendix C shows the data used for the material analyzed. Table 9 shows the range of variables input into a computing program called MINITAB.

Table 9. Maximum and minimum metallic values for modulus, density and beam width.

	Minimum	Maximum
Modulus of Elasticity (*10^6 psi)	6.5 (Magnesium)	58 (Tungsten)
Density (lb/in^3)	0.0639 (Magnesium)	0.697 (Gold)
Beam width (in)	0.25"	1.0"

C. MINITAB

MINITAB is a systems engineering analysis tool that takes a given set of inputs and returns desired outputs. For this analysis, the inputs are to reduce the area of material in the modifiable section, change the material to something other than Aluminum, or a combination of the two. The outputs of the MINITAB program are the new mass of the changed wall and the corresponding stiffness. These are the same outputs seen from MATLAB. However, an improvement of MINITAB over MATLAB is its ability to analyze three or more parameters simultaneously. In essence, a three-dimensional graph can be analyzed by holding one of the variables and looking at how the other two variables respond to the held value. This is the Response Surface Model (RSM). In the RSM, the feasibility solution space is identified. The structure is not to exceed 177 lbs. The output graphs will show regions of acceptable material Modulus of Elasticity and density. From here, a material only needs to be selected that fits these criteria to satisfy the constraints.

Using the information from Table 9, a list of 20 random combinations is created. This data is then input into MATLAB using configuration F program code. The maximum stress and displacements are found and in conjunction with the calculated mass, the input is ready for MINITAB. This data

is shown in Appendix D. Once the mass and stiffness data is entered, the program generates the region of acceptance in the solution. Figure 31 shows the constraint of the data entered into MINITAB.

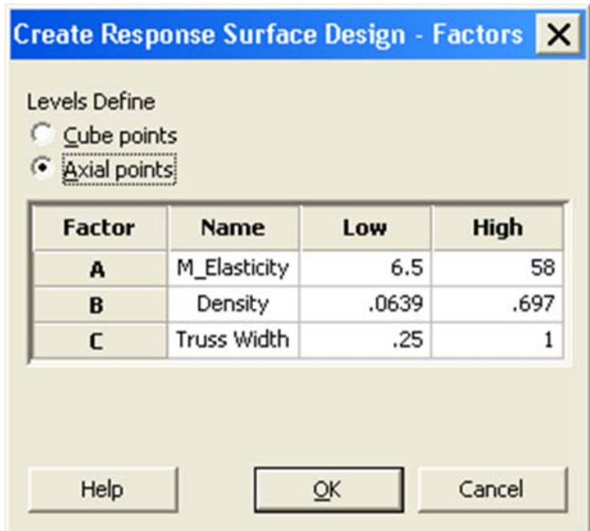

Figure 31. Parameters used for MINITAB input.

The outputs of looking at the desired 0.25-, 0.5- and 1-inch beam width elements are shown in Figures 32-34, respectively. In Figure 32, where the beam width is 0.25 inches, the acceptability region is located in the white section in the upper left-hand corner. Flagged on the graph are approximately where the metals Titanium (density of 0.163 lb/in^3), Aluminum (density of 0.101 lb/in^3) and Magnesium (density of 0.0639) would be located. These are the three lowest density materials looked at and, for 0.25-inch thickness of beams, do not satisfy the mass and or stiffness constraints. Figure 33 has a larger feasibility region, but the materials still do not satisfy the constraints. In Figure 34, the feasibility region shows that the material needs to have a density of less than 0.108 lbs/in^3 and a Modulus of

Elasticity between 9.25 x 10^6 psi and 40.83 x 10^6 psi. Aluminum and Aluminum only, becomes an acceptable solution. This hereby reconfirms the design already established in previous chapters that Aluminum 7075 should be used in Configuration F with a beam width of 1 inch. The units of M_Elasticity are $*10^6$ psi and the units for density are lb/in^3.

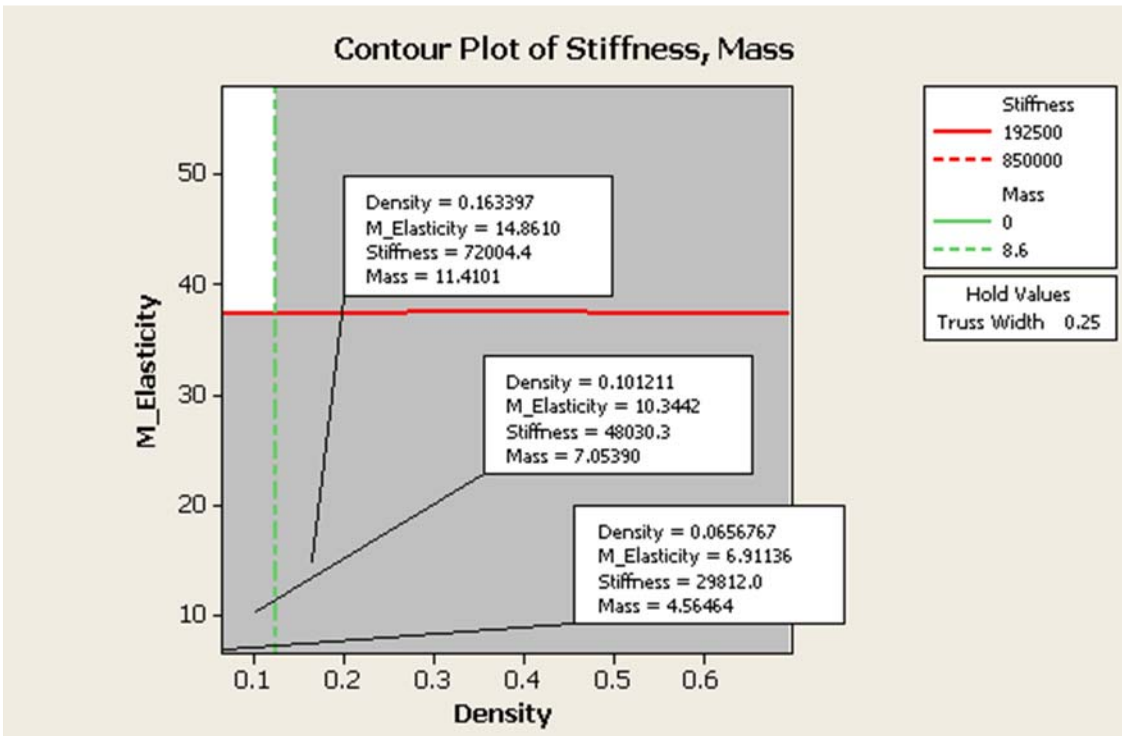

Figure 32. MINITAB results of holding the beam width at 0.25 inches.

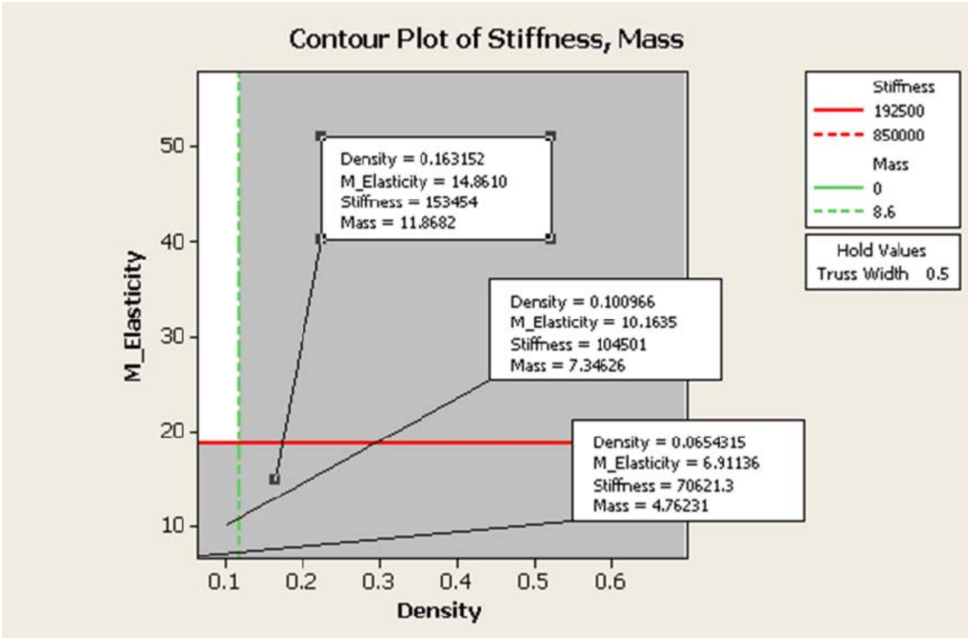

Figure 33. MINITAB results of holding the beam width at 0.5 inches.

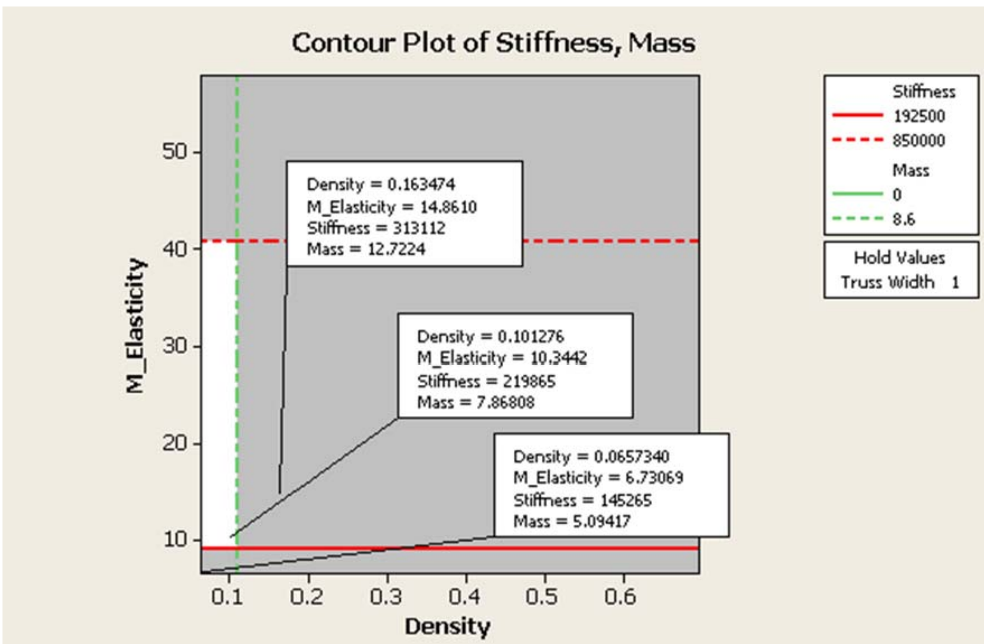

Figure 34. MINITAB results of holding the beam width at 1 inch.

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VI. COMPOUND LOAD ANALYSIS

A. ADDITIONAL ANALYSIS

Now that the optimum material is determined to be space-grade Aluminum and the optimum configuration is the pattern designated by F, the next step can be taken. A thorough analysis needs to be done on Configuration F. The first analysis was conducted with the gravitational forces being applied in the negative Y direction. However, for this two-dimensional analysis, the structure also needs to be analyzed for forces in the positive and negative X directions, the positive Y direction, and a combination of the two. Figures 35-38 show all force combinations analyzed.

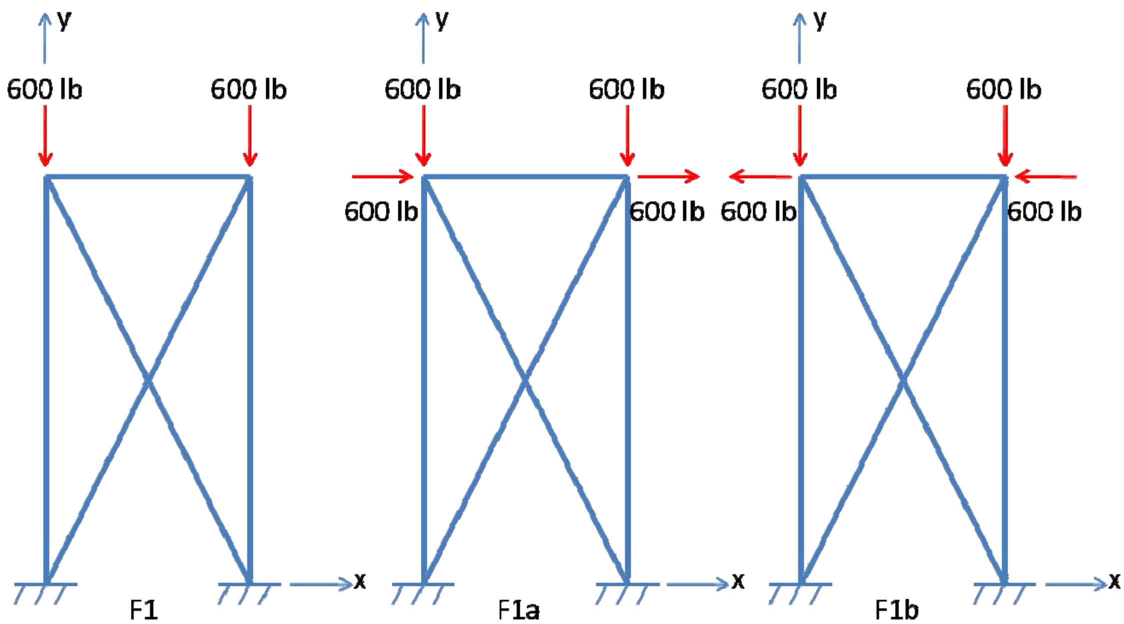

Figure 35. Diagrams of compound forces analyzed (F1–F1b).

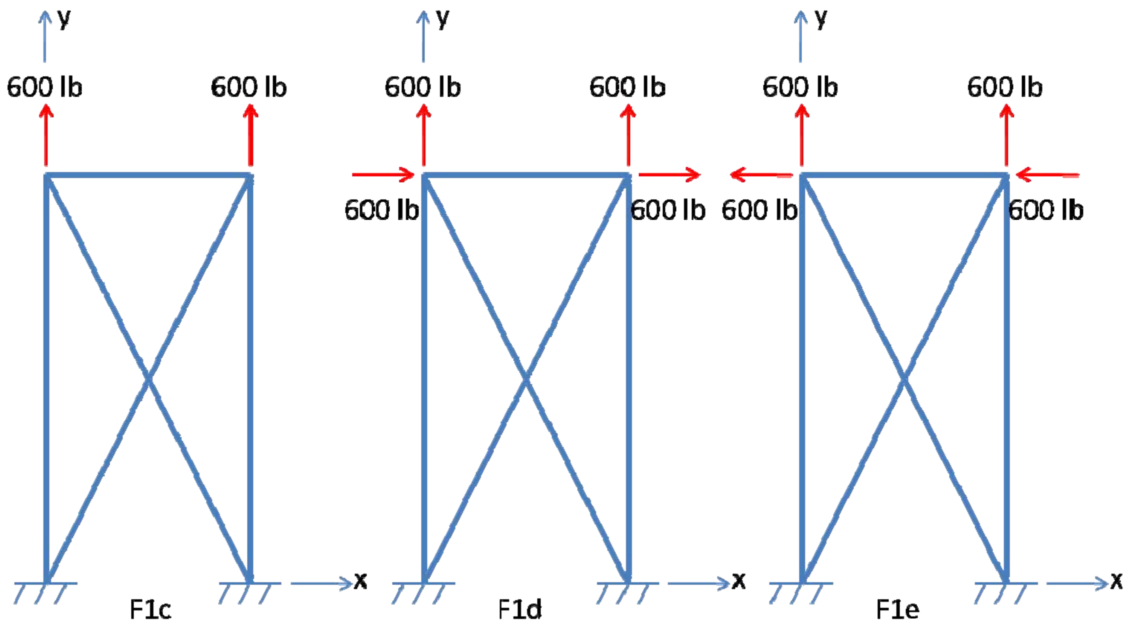

Figure 36. Diagrams of compound forces analyzed (F1c–F1e).

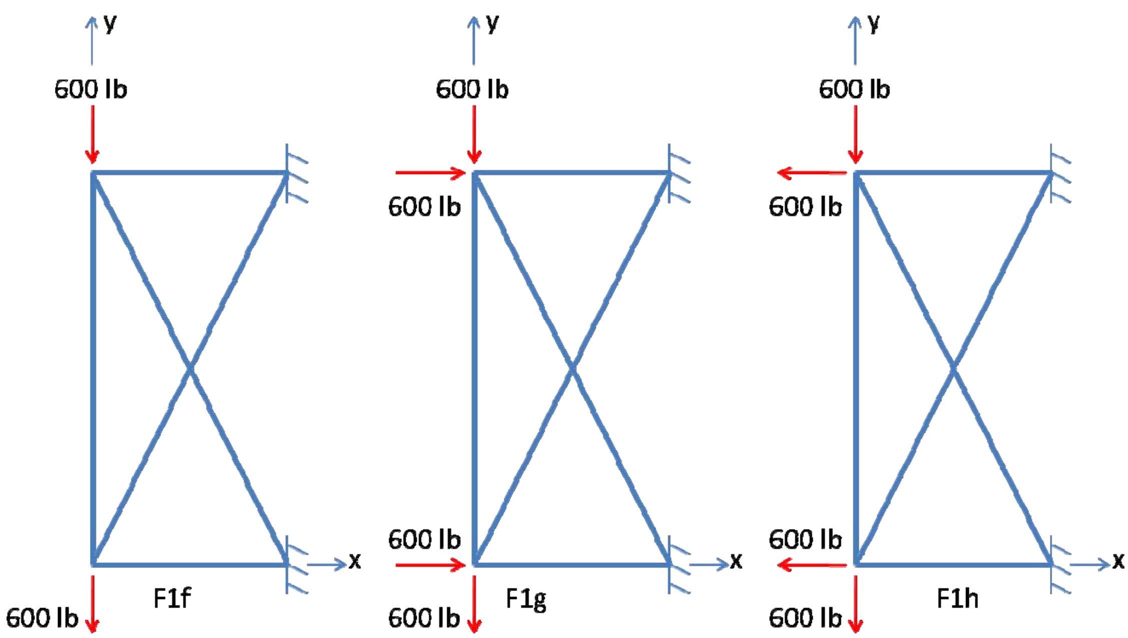

Figure 37. Diagrams of compound forces analyzed (F1f–F1h).

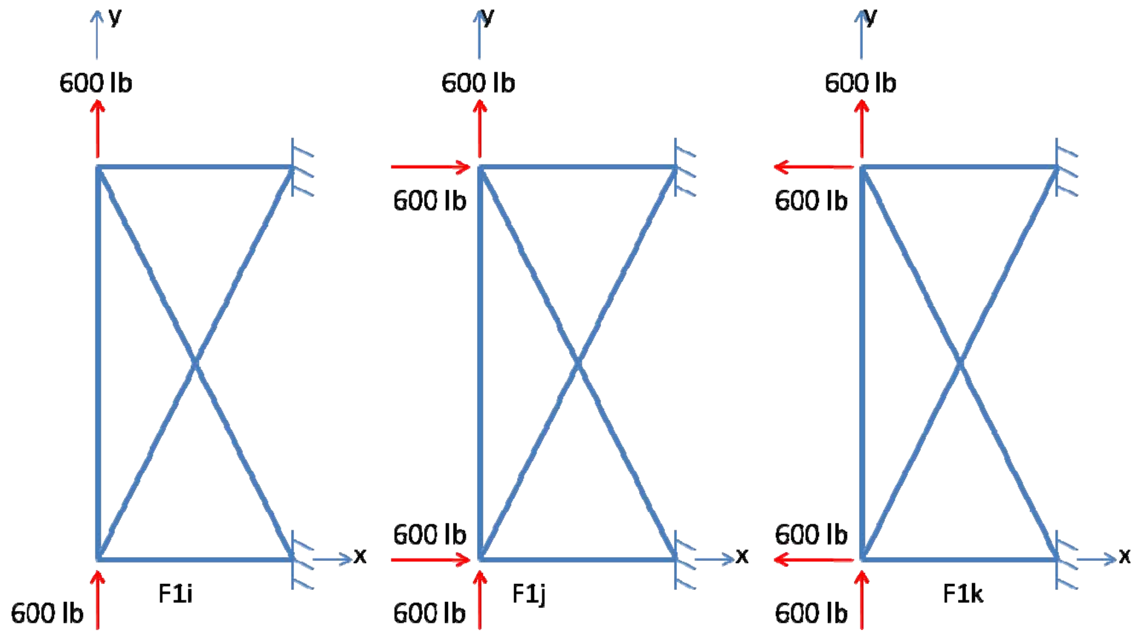

Figure 38. Diagrams of compound forces analyzed (F1i–F1k).

B. MATLAB RESULTS

The compound loads from Figures 35-38 are put into MATLAB to return the maximum stress and deflections for each design. Appendix E contains the MATLAB code used for analysis, and Appendix F shows the stress and displacements for each element and node. The maximum values were then used in Appendix G to solve for how each combined load affects the reduction in stiffness. Figure 39 shows each setup's corresponding stress and deflection. From this chart, it can be seen that the largest deflection and stress is seen in setups a, b, d and e. Figure 40 shows how each setup compares in stiffness percentages. The ultimate stress of Aluminum 7075 is 68 ksi, and the yield stress is 57 ksi. The maximum setup stress is 6.2 ksi, which is vastly smaller than the limits of the material.

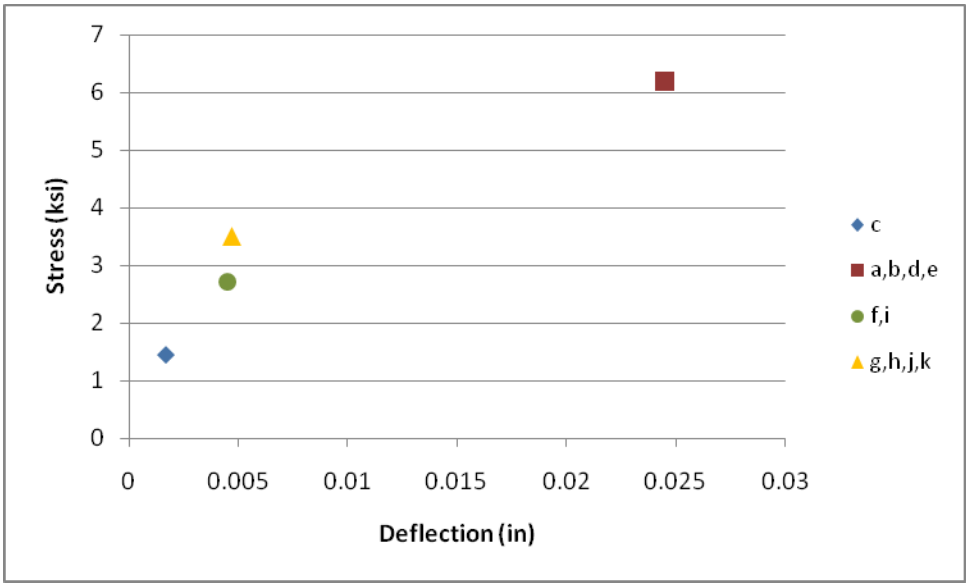

Figure 39. Stress and deflections for all setups.

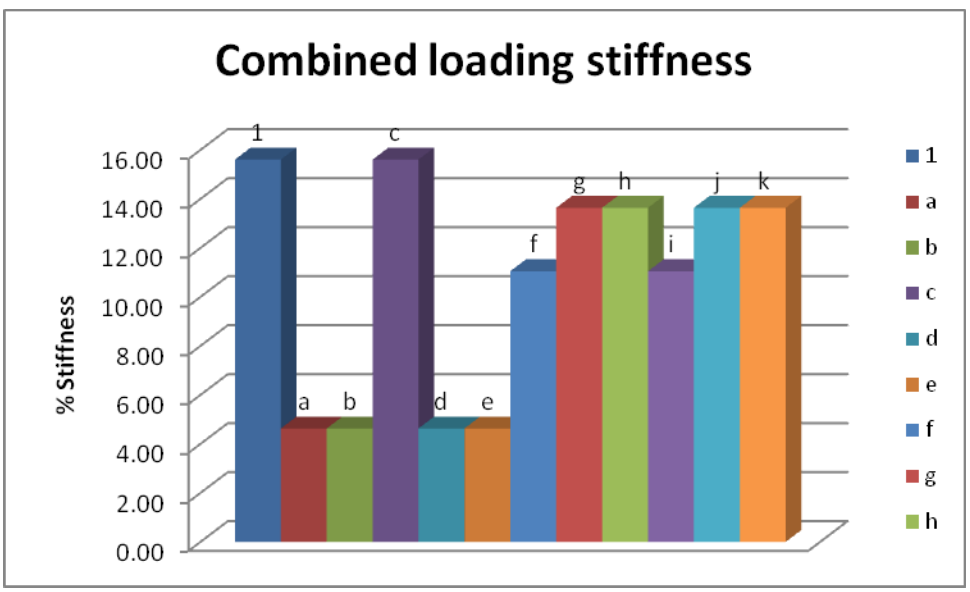

Figure 40. Percent reduction in stiffness variations for compound loadings.

VII. CONCLUSION

A. CONFIGURATION

Mass is the leading cause of high cost prices for the construction and launch of satellites. Something with a lot of mass costs more in materials and takes more time to integrate all the systems. This integration also contributes a large portion to the full cost. As a result, the goal for any satellite is to have minimal mass. The NPSCuL structure carries up to 24 CubeSats into space. With the decrease in size of components for these satellites, the CubeSat designers are able to put more and more mass into the 10 cm^3 volume. These designers constantly desire more and more mass to be allowed. Optimizing the weight of NPSCuL can either allow more mass to be put into the CubeSats, or can go toward attempting to lower the magnification of forces seen at resonance frequencies during launch of the rocket that carries the structure to space. Looking at ways to optimize mass utilization of modifications to the modifiable area is ascertained. Looking at various truss like structures contained inside the MA to remove as much mass as possible, while maintaining some stiffness, was completed. Using an “X” shape of crossbeams in the desired area proved to be the best design.

B. ALUMINUM 7075 AND COMBINED LOADING

The NPSCuL structure does not have to operate in space for the designed life that a satellite does. It is merely the transportation of the satellites on their way to space.

As such, the next step in optimization was to evaluate whether Aluminum 7075 (space-grade Aluminum) was the best choice for materials. Analyzing the range of metals from Magnesium to Tungsten to Gold, a simultaneous analysis was conducted using the “X”-shaped configuration. The percent reduction in stiffness and the percent reduction in mass were evaluated at different crossbeam widths for all the metals using an analysis program called MINITAB. The constraint is that the structure cannot exceed 177 lbs. It was found that the crossbeam width must be 1 inch and the material must be an Aluminum alloy to meet the requirements.

The initial analysis used only forces in one direction for calculations and comparisons. The next step to evaluate was if the compound loading, which the structure will see, renders the design a failure. The yield stress of Aluminum 7075 is 57 ksi. A combination of directions and forces were applied and the maximum stress seen in any of the members was 6.2 ksi.

C. FUTURE WORK

This optimization of mass looked at the two-dimensional response of the separate walls. Each wall only had six ksi of a stress when analyzed with compound loads and the yield of the material is 57. The ultimate of the material is 68 ksi and stresses seen were ten times less than that. The next step in the process will be to take the modifications in the modifiable area and construct a model in IDEAS. From there, a dynamic response can be determined; if it is a plausible solution, the structure should be built and vibration tested. The analysis of nonmetal options for the structure should also be undertaken.

APPENDIX A

FEMA1

```
nel=4;
nnel=2;
ndof=2;
nnode=4;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=4;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=4;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(4)=-600;
ff(6)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
```

```
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end
```

```
    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMA5

```
nel=4;
nnel=2;
ndof=2;
nnode=4;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;

elprop(1,1)=10300000;
elprop(1,2)=.125;
elprop(2,1)=10300000;
elprop(2,2)=.125;
elprop(3,1)=10300000;
elprop(3,2)=.125;
elprop(4,1)=10300000;
elprop(4,2)=.125;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=4;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=4;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(4)=-600;
ff(6)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
```

```
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;
```

```
    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMA25

```
nel=4;
nnel=2;
ndof=2;
nnode=4;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;

elprop(1,1)=10300000;
elprop(1,2)=.0625;
elprop(2,1)=10300000;
elprop(2,2)=.0625;
elprop(3,1)=10300000;
elprop(3,2)=.0625;
elprop(4,1)=10300000;
elprop(4,2)=.0625;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=4;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=4;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(4)=-600;
ff(6)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
```

```
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;
```

```
    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMB1

```
nel=8;
nnel=2;
ndof=2;
nnode=6;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;
elprop(8,1)=10300000;
elprop(8,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=6;
nodes(3,1)=2;nodes(3,2)=5;
nodes(4,1)=2;nodes(4,2)=3;
nodes(5,1)=3;nodes(5,2)=5;
nodes(6,1)=3;nodes(6,2)=4;
nodes(7,1)=4;nodes(7,2)=5;
nodes(8,1)=5;nodes(8,2)=6;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
```

```
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
```

```
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMB5

```
nel=8;
nnel=2;
ndof=2;
nnode=6;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;

elprop(1,1)=10300000;
elprop(1,2)=.125;
elprop(2,1)=10300000;
elprop(2,2)=.125;
elprop(3,1)=10300000;
elprop(3,2)=.125;
elprop(4,1)=10300000;
elprop(4,2)=.125;
elprop(5,1)=10300000;
elprop(5,2)=.125;
elprop(6,1)=10300000;
elprop(6,2)=.125;
elprop(7,1)=10300000;
elprop(7,2)=.125;
elprop(8,1)=10300000;
elprop(8,2)=.125;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=6;
nodes(3,1)=2;nodes(3,2)=5;
nodes(4,1)=2;nodes(4,2)=3;
nodes(5,1)=3;nodes(5,2)=5;
nodes(6,1)=3;nodes(6,2)=4;
nodes(7,1)=4;nodes(7,2)=5;
nodes(8,1)=5;nodes(8,2)=6;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
```

```
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
```

```
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMB25

```
nel=8;
nnel=2;
ndof=2;
nnode=6;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;

elprop(1,1)=10300000;
elprop(1,2)=.0625;
elprop(2,1)=10300000;
elprop(2,2)=.0625;
elprop(3,1)=10300000;
elprop(3,2)=.0625;
elprop(4,1)=10300000;
elprop(4,2)=.0625;
elprop(5,1)=10300000;
elprop(5,2)=.0625;
elprop(6,1)=10300000;
elprop(6,2)=.0625;
elprop(7,1)=10300000;
elprop(7,2)=.0625;
elprop(8,1)=10300000;
elprop(8,2)=.0625;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=6;
nodes(3,1)=2;nodes(3,2)=5;
nodes(4,1)=2;nodes(4,2)=3;
nodes(5,1)=3;nodes(5,2)=5;
nodes(6,1)=3;nodes(6,2)=4;
nodes(7,1)=4;nodes(7,2)=5;
nodes(8,1)=5;nodes(8,2)=6;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
```

```
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
```

```
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMC1

```
nel=8;
nnel=2;
ndof=2;
nnode=6;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;
elprop(8,1)=10300000;
elprop(8,2)=.25;

nodes(1,1)=1;nodes(1,2)=5;
nodes(2,1)=1;nodes(2,2)=2;
nodes(3,1)=2;nodes(3,2)=5;
nodes(4,1)=2;nodes(4,2)=3;
nodes(5,1)=3;nodes(5,2)=5;
nodes(6,1)=3;nodes(6,2)=4;
nodes(7,1)=4;nodes(7,2)=5;
nodes(8,1)=5;nodes(8,2)=6;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
```

```
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
```

```
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMC5

```
nel=8;
nnel=2;
ndof=2;
nnode=6;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;

elprop(1,1)=10300000;
elprop(1,2)=.125;
elprop(2,1)=10300000;
elprop(2,2)=.125;
elprop(3,1)=10300000;
elprop(3,2)=.125;
elprop(4,1)=10300000;
elprop(4,2)=.125;
elprop(5,1)=10300000;
elprop(5,2)=.125;
elprop(6,1)=10300000;
elprop(6,2)=.125;
elprop(7,1)=10300000;
elprop(7,2)=.125;
elprop(8,1)=10300000;
elprop(8,2)=.125;

nodes(1,1)=1;nodes(1,2)=5;
nodes(2,1)=1;nodes(2,2)=2;
nodes(3,1)=2;nodes(3,2)=5;
nodes(4,1)=2;nodes(4,2)=3;
nodes(5,1)=3;nodes(5,2)=5;
nodes(6,1)=3;nodes(6,2)=4;
nodes(7,1)=4;nodes(7,2)=5;
nodes(8,1)=5;nodes(8,2)=6;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
```

```
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
```

```
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMC25

```
nel=8;
nnel=2;
ndof=2;
nnode=6;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;

elprop(1,1)=10300000;
elprop(1,2)=.0625;
elprop(2,1)=10300000;
elprop(2,2)=.0625;
elprop(3,1)=10300000;
elprop(3,2)=.0625;
elprop(4,1)=10300000;
elprop(4,2)=.0625;
elprop(5,1)=10300000;
elprop(5,2)=.0625;
elprop(6,1)=10300000;
elprop(6,2)=.0625;
elprop(7,1)=10300000;
elprop(7,2)=.0625;
elprop(8,1)=10300000;
elprop(8,2)=.0625;

nodes(1,1)=1;nodes(1,2)=5;
nodes(2,1)=1;nodes(2,2)=2;
nodes(3,1)=2;nodes(3,2)=5;
nodes(4,1)=2;nodes(4,2)=3;
nodes(5,1)=3;nodes(5,2)=5;
nodes(6,1)=3;nodes(6,2)=4;
nodes(7,1)=4;nodes(7,2)=5;
nodes(8,1)=5;nodes(8,2)=6;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
```

```
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
```

```
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMD1

```
nel=11;
nnel=2;
ndof=2;
nnode=7;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;
gcoord(7,1)=3.1825;gcoord(7,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;
elprop(8,1)=10300000;
elprop(8,2)=.25;
elprop(9,1)=10300000;
elprop(9,2)=.25;
elprop(10,1)=10300000;
elprop(10,2)=.25;
elprop(11,1)=10300000;
elprop(11,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=7;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=7;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=7;
nodes(7,1)=4;nodes(7,2)=5;
nodes(8,1)=5;nodes(8,2)=7;
nodes(9,1)=5;nodes(9,2)=6;
nodes(10,1)=6;nodes(10,2)=7;
nodes(11,1)=7;nodes(11,2)=1;

bcdof(1)=1;
bcval(1)=0;
```

```
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);
```

```
    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMD5

```
nel=11;
nnel=2;
ndof=2;
nnode=7;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;
gcoord(7,1)=3.1825;gcoord(7,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.125;
elprop(2,1)=10300000;
elprop(2,2)=.125;
elprop(3,1)=10300000;
elprop(3,2)=.125;
elprop(4,1)=10300000;
elprop(4,2)=.125;
elprop(5,1)=10300000;
elprop(5,2)=.125;
elprop(6,1)=10300000;
elprop(6,2)=.125;
elprop(7,1)=10300000;
elprop(7,2)=.125;
elprop(8,1)=10300000;
elprop(8,2)=.125;
elprop(9,1)=10300000;
elprop(9,2)=.125;
elprop(10,1)=10300000;
elprop(10,2)=.125;
elprop(11,1)=10300000;
elprop(11,2)=.125;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=7;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=7;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=7;
nodes(7,1)=4;nodes(7,2)=5;
nodes(8,1)=5;nodes(8,2)=7;
nodes(9,1)=5;nodes(9,2)=6;
nodes(10,1)=6;nodes(10,2)=7;
nodes(11,1)=7;nodes(11,2)=1;

bcdof(1)=1;
bcval(1)=0;
```

```
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);
```

```
    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMD25

```
nel=11;
nnel=2;
ndof=2;
nnode=7;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;
gcoord(7,1)=3.1825;gcoord(7,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.0625;
elprop(2,1)=10300000;
elprop(2,2)=.0625;
elprop(3,1)=10300000;
elprop(3,2)=.0625;
elprop(4,1)=10300000;
elprop(4,2)=.0625;
elprop(5,1)=10300000;
elprop(5,2)=.0625;
elprop(6,1)=10300000;
elprop(6,2)=.0625;
elprop(7,1)=10300000;
elprop(7,2)=.0625;
elprop(8,1)=10300000;
elprop(8,2)=.0625;
elprop(9,1)=10300000;
elprop(9,2)=.0625;
elprop(10,1)=10300000;
elprop(10,2)=.0625;
elprop(11,1)=10300000;
elprop(11,2)=.0625;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=7;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=7;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=7;
nodes(7,1)=4;nodes(7,2)=5;
nodes(8,1)=5;nodes(8,2)=7;
nodes(9,1)=5;nodes(9,2)=6;
nodes(10,1)=6;nodes(10,2)=7;
nodes(11,1)=7;nodes(11,2)=1;

bcdof(1)=1;
bcval(1)=0;
```

```
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);
```

```
    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEME1

```
nel=12;
nnel=2;
ndof=2;
nnode=8;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=3.1825;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=12;
gcoord(6,1)=6.365;gcoord(6,2)=6;
gcoord(7,1)=6.365;gcoord(7,2)=0;
gcoord(8,1)=3.1825;gcoord(8,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;
elprop(8,1)=10300000;
elprop(8,2)=.25;
elprop(9,1)=10300000;
elprop(9,2)=.25;
elprop(10,1)=10300000;
elprop(10,2)=.25;
elprop(11,1)=10300000;
elprop(11,2)=.25;
elprop(12,1)=10300000;
elprop(12,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=8;
nodes(3,1)=2;nodes(3,2)=4;
nodes(4,1)=2;nodes(4,2)=3;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=6;
nodes(7,1)=4;nodes(7,2)=5;
nodes(8,1)=5;nodes(8,2)=6;
nodes(9,1)=6;nodes(9,2)=8;
nodes(10,1)=6;nodes(10,2)=7;
nodes(11,1)=7;nodes(11,2)=8;
```

```
nodes(12,1)=8;nodes(12,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=13;
bcval(3)=0;
bcdof(4)=14;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-400;
ff(8)=-400;
ff(10)=-400;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;
```

```
for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEME5

```
nel=12;
nnel=2;
ndof=2;
nnode=8;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=3.1825;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=12;
gcoord(6,1)=6.365;gcoord(6,2)=6;
gcoord(7,1)=6.365;gcoord(7,2)=0;
gcoord(8,1)=3.1825;gcoord(8,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.125;
elprop(2,1)=10300000;
elprop(2,2)=.125;
elprop(3,1)=10300000;
elprop(3,2)=.125;
elprop(4,1)=10300000;
elprop(4,2)=.125;
elprop(5,1)=10300000;
elprop(5,2)=.125;
elprop(6,1)=10300000;
elprop(6,2)=.125;
elprop(7,1)=10300000;
elprop(7,2)=.125;
elprop(8,1)=10300000;
elprop(8,2)=.125;
elprop(9,1)=10300000;
elprop(9,2)=.125;
elprop(10,1)=10300000;
elprop(10,2)=.125;
elprop(11,1)=10300000;
elprop(11,2)=.125;
elprop(12,1)=10300000;
elprop(12,2)=.125;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=8;
nodes(3,1)=2;nodes(3,2)=4;
nodes(4,1)=2;nodes(4,2)=3;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=6;
nodes(7,1)=4;nodes(7,2)=5;
nodes(8,1)=5;nodes(8,2)=6;
nodes(9,1)=6;nodes(9,2)=8;
nodes(10,1)=6;nodes(10,2)=7;
nodes(11,1)=7;nodes(11,2)=8;
```

```
nodes(12,1)=8;nodes(12,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=13;
bcval(3)=0;
bcdof(4)=14;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-400;
ff(8)=-400;
ff(10)=-400;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;
```

```
for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEME25

```
nel=12;
nnel=2;
ndof=2;
nnode=8;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=3.1825;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=12;
gcoord(6,1)=6.365;gcoord(6,2)=6;
gcoord(7,1)=6.365;gcoord(7,2)=0;
gcoord(8,1)=3.1825;gcoord(8,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.0625;
elprop(2,1)=10300000;
elprop(2,2)=.0625;
elprop(3,1)=10300000;
elprop(3,2)=.0625;
elprop(4,1)=10300000;
elprop(4,2)=.0625;
elprop(5,1)=10300000;
elprop(5,2)=.0625;
elprop(6,1)=10300000;
elprop(6,2)=.0625;
elprop(7,1)=10300000;
elprop(7,2)=.0625;
elprop(8,1)=10300000;
elprop(8,2)=.0625;
elprop(9,1)=10300000;
elprop(9,2)=.0625;
elprop(10,1)=10300000;
elprop(10,2)=.0625;
elprop(11,1)=10300000;
elprop(11,2)=.0625;
elprop(12,1)=10300000;
elprop(12,2)=.0625;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=8;
nodes(3,1)=2;nodes(3,2)=4;
nodes(4,1)=2;nodes(4,2)=3;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=6;
nodes(7,1)=4;nodes(7,2)=5;
nodes(8,1)=5;nodes(8,2)=6;
nodes(9,1)=6;nodes(9,2)=8;
nodes(10,1)=6;nodes(10,2)=7;
nodes(11,1)=7;nodes(11,2)=8;
```

```
nodes(12,1)=8;nodes(12,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=13;
bcval(3)=0;
bcdof(4)=14;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-400;
ff(8)=-400;
ff(10)=-400;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;
```

```
for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF1

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=5;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(4)=-600;
ff(6)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
```

```
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF5

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.125;
elprop(2,1)=10300000;
elprop(2,2)=.125;
elprop(3,1)=10300000;
elprop(3,2)=.125;
elprop(4,1)=10300000;
elprop(4,2)=.125;
elprop(5,1)=10300000;
elprop(5,2)=.125;
elprop(6,1)=10300000;
elprop(6,2)=.125;
elprop(7,1)=10300000;
elprop(7,2)=.125;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=5;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(4)=-600;
ff(6)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
```

```
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF25

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.0625;
elprop(2,1)=10300000;
elprop(2,2)=.0625;
elprop(3,1)=10300000;
elprop(3,2)=.0625;
elprop(4,1)=10300000;
elprop(4,2)=.0625;
elprop(5,1)=10300000;
elprop(5,2)=.0625;
elprop(6,1)=10300000;
elprop(6,2)=.0625;
elprop(7,1)=10300000;
elprop(7,2)=.0625;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=5;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(4)=-600;
ff(6)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
```

```
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMG1

```
nel=13;
nnel=2;
ndof=2;
nnode=8;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;
gcoord(7,1)=3.1825;gcoord(7,2)=3;
gcoord(8,1)=3.1825;gcoord(8,2)=9;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;
elprop(8,1)=10300000;
elprop(8,2)=.25;
elprop(9,1)=10300000;
elprop(9,2)=.25;
elprop(10,1)=10300000;
elprop(10,2)=.25;
elprop(11,1)=10300000;
elprop(11,2)=.25;
elprop(12,1)=10300000;
elprop(12,2)=.25;
elprop(13,1)=10300000;
elprop(13,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=7;
nodes(3,1)=2;nodes(3,2)=8;
nodes(4,1)=2;nodes(4,2)=3;
nodes(5,1)=3;nodes(5,2)=8;
nodes(6,1)=3;nodes(6,2)=4;
nodes(7,1)=4;nodes(7,2)=8;
nodes(8,1)=4;nodes(8,2)=5;
nodes(9,1)=5;nodes(9,2)=8;
```

```
nodes(10,1)=5;nodes(10,2)=7;
nodes(11,1)=5;nodes(11,2)=6;
nodes(12,1)=6;nodes(12,2)=7;
nodes(13,1)=7;nodes(13,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;
```

```
for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMG5

```
nel=13;
nnel=2;
ndof=2;
nnode=8;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;
gcoord(7,1)=3.1825;gcoord(7,2)=3;
gcoord(8,1)=3.1825;gcoord(8,2)=9;

elprop(1,1)=10300000;
elprop(1,2)=.125;
elprop(2,1)=10300000;
elprop(2,2)=.125;
elprop(3,1)=10300000;
elprop(3,2)=.125;
elprop(4,1)=10300000;
elprop(4,2)=.125;
elprop(5,1)=10300000;
elprop(5,2)=.125;
elprop(6,1)=10300000;
elprop(6,2)=.125;
elprop(7,1)=10300000;
elprop(7,2)=.125;
elprop(8,1)=10300000;
elprop(8,2)=.125;
elprop(9,1)=10300000;
elprop(9,2)=.125;
elprop(10,1)=10300000;
elprop(10,2)=.125;
elprop(11,1)=10300000;
elprop(11,2)=.125;
elprop(12,1)=10300000;
elprop(12,2)=.125;
elprop(13,1)=10300000;
elprop(13,2)=.125;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=7;
nodes(3,1)=2;nodes(3,2)=8;
nodes(4,1)=2;nodes(4,2)=3;
nodes(5,1)=3;nodes(5,2)=8;
nodes(6,1)=3;nodes(6,2)=4;
nodes(7,1)=4;nodes(7,2)=8;
nodes(8,1)=4;nodes(8,2)=5;
nodes(9,1)=5;nodes(9,2)=8;
```

```
nodes(10,1)=5;nodes(10,2)=7;
nodes(11,1)=5;nodes(11,2)=6;
nodes(12,1)=6;nodes(12,2)=7;
nodes(13,1)=7;nodes(13,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;
```

```
for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMG25

```
nel=13;
nnel=2;
ndof=2;
nnode=8;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;
gcoord(7,1)=3.1825;gcoord(7,2)=3;
gcoord(8,1)=3.1825;gcoord(8,2)=9;

elprop(1,1)=10300000;
elprop(1,2)=.0625;
elprop(2,1)=10300000;
elprop(2,2)=.0625;
elprop(3,1)=10300000;
elprop(3,2)=.0625;
elprop(4,1)=10300000;
elprop(4,2)=.0625;
elprop(5,1)=10300000;
elprop(5,2)=.0625;
elprop(6,1)=10300000;
elprop(6,2)=.0625;
elprop(7,1)=10300000;
elprop(7,2)=.0625;
elprop(8,1)=10300000;
elprop(8,2)=.0625;
elprop(9,1)=10300000;
elprop(9,2)=.0625;
elprop(10,1)=10300000;
elprop(10,2)=.0625;
elprop(11,1)=10300000;
elprop(11,2)=.0625;
elprop(12,1)=10300000;
elprop(12,2)=.0625;
elprop(13,1)=10300000;
elprop(13,2)=.0625;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=7;
nodes(3,1)=2;nodes(3,2)=8;
nodes(4,1)=2;nodes(4,2)=3;
nodes(5,1)=3;nodes(5,2)=8;
nodes(6,1)=3;nodes(6,2)=4;
nodes(7,1)=4;nodes(7,2)=8;
nodes(8,1)=4;nodes(8,2)=5;
nodes(9,1)=5;nodes(9,2)=8;
```

```
nodes(10,1)=5;nodes(10,2)=7;
nodes(11,1)=5;nodes(11,2)=6;
nodes(12,1)=6;nodes(12,2)=7;
nodes(13,1)=7;nodes(13,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;
```

```
for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMH1

```
nel=14;
nnel=2;
ndof=2;
nnode=8;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;
gcoord(7,1)=3.1825;gcoord(7,2)=3;
gcoord(8,1)=3.1825;gcoord(8,2)=9;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;
elprop(8,1)=10300000;
elprop(8,2)=.25;
elprop(9,1)=10300000;
elprop(9,2)=.25;
elprop(10,1)=10300000;
elprop(10,2)=.25;
elprop(11,1)=10300000;
elprop(11,2)=.25;
elprop(12,1)=10300000;
elprop(12,2)=.25;
elprop(13,1)=10300000;
elprop(13,2)=.25;
elprop(14,1)=10300000;
elprop(14,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=7;
nodes(3,1)=2;nodes(3,2)=5;
nodes(4,1)=2;nodes(4,2)=8;
nodes(5,1)=2;nodes(5,2)=3;
nodes(6,1)=3;nodes(6,2)=8;
nodes(7,1)=3;nodes(7,2)=4;
```

```
nodes(8,1)=4;nodes(8,2)=8;
nodes(9,1)=4;nodes(9,2)=5;
nodes(10,1)=5;nodes(10,2)=8;
nodes(11,1)=5;nodes(11,2)=7;
nodes(12,1)=5;nodes(12,2)=6;
nodes(13,1)=6;nodes(13,2)=7;
nodes(14,1)=7;nodes(14,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end
```

```
[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMH5

```
nel=14;
nnel=2;
ndof=2;
nnode=8;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;
gcoord(7,1)=3.1825;gcoord(7,2)=3;
gcoord(8,1)=3.1825;gcoord(8,2)=9;

elprop(1,1)=10300000;
elprop(1,2)=.125;
elprop(2,1)=10300000;
elprop(2,2)=.125;
elprop(3,1)=10300000;
elprop(3,2)=.125;
elprop(4,1)=10300000;
elprop(4,2)=.125;
elprop(5,1)=10300000;
elprop(5,2)=.125;
elprop(6,1)=10300000;
elprop(6,2)=.125;
elprop(7,1)=10300000;
elprop(7,2)=.125;
elprop(8,1)=10300000;
elprop(8,2)=.125;
elprop(9,1)=10300000;
elprop(9,2)=.125;
elprop(10,1)=10300000;
elprop(10,2)=.125;
elprop(11,1)=10300000;
elprop(11,2)=.125;
elprop(12,1)=10300000;
elprop(12,2)=.125;
elprop(13,1)=10300000;
elprop(13,2)=.125;
elprop(14,1)=10300000;
elprop(14,2)=.125;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=7;
nodes(3,1)=2;nodes(3,2)=5;
nodes(4,1)=2;nodes(4,2)=8;
nodes(5,1)=2;nodes(5,2)=3;
nodes(6,1)=3;nodes(6,2)=8;
nodes(7,1)=3;nodes(7,2)=4;
```

```
nodes(8,1)=4;nodes(8,2)=8;
nodes(9,1)=4;nodes(9,2)=5;
nodes(10,1)=5;nodes(10,2)=8;
nodes(11,1)=5;nodes(11,2)=7;
nodes(12,1)=5;nodes(12,2)=6;
nodes(13,1)=6;nodes(13,2)=7;
nodes(14,1)=7;nodes(14,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end
```

```
[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMH25

```
nel=14;
nnel=2;
ndof=2;
nnode=8;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=6;
gcoord(3,1)=0;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=12;
gcoord(5,1)=6.365;gcoord(5,2)=6;
gcoord(6,1)=6.365;gcoord(6,2)=0;
gcoord(7,1)=3.1825;gcoord(7,2)=3;
gcoord(8,1)=3.1825;gcoord(8,2)=9;

elprop(1,1)=10300000;
elprop(1,2)=.0625;
elprop(2,1)=10300000;
elprop(2,2)=.0625;
elprop(3,1)=10300000;
elprop(3,2)=.0625;
elprop(4,1)=10300000;
elprop(4,2)=.0625;
elprop(5,1)=10300000;
elprop(5,2)=.0625;
elprop(6,1)=10300000;
elprop(6,2)=.0625;
elprop(7,1)=10300000;
elprop(7,2)=.0625;
elprop(8,1)=10300000;
elprop(8,2)=.0625;
elprop(9,1)=10300000;
elprop(9,2)=.0625;
elprop(10,1)=10300000;
elprop(10,2)=.0625;
elprop(11,1)=10300000;
elprop(11,2)=.0625;
elprop(12,1)=10300000;
elprop(12,2)=.0625;
elprop(13,1)=10300000;
elprop(13,2)=.0625;
elprop(14,1)=10300000;
elprop(14,2)=.0625;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=7;
nodes(3,1)=2;nodes(3,2)=5;
nodes(4,1)=2;nodes(4,2)=8;
nodes(5,1)=2;nodes(5,2)=3;
nodes(6,1)=3;nodes(6,2)=8;
nodes(7,1)=3;nodes(7,2)=4;
```

```
nodes(8,1)=4;nodes(8,2)=8;
nodes(9,1)=4;nodes(9,2)=5;
nodes(10,1)=5;nodes(10,2)=8;
nodes(11,1)=5;nodes(11,2)=7;
nodes(12,1)=5;nodes(12,2)=6;
nodes(13,1)=6;nodes(13,2)=7;
nodes(14,1)=7;nodes(14,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=11;
bcval(3)=0;
bcdof(4)=12;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(6)=-600;
ff(8)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end
```

```
[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

APPENDIX B

H	0.25"	H	0.5"	H	1"	G	0.25"	G	0.5"	G	1"	E	0.25"	E	0.5"	E	1"
displ =		displ =		displ =		displ =		displ =		displ =		displ =		displ =		displ =	
1.0000	-0.0000	1.0000	-0.0000	1.0000	-0.0000	1.0000	-0.0000	1.0000	-0.0000	1.0000	-0.0000	1.0000	-0.0000	1.0000	-0.0000	1.0000	-0.0000
2.0000	-0.0000	2.0000	-0.0000	2.0000	-0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000
3.0000	-0.0011	3.0000	-0.0006	3.0000	-0.0003	3.0000	-0.0053	3.0000	-0.0026	3.0000	-0.0013	3.0000	-0.0005	3.0000	-0.0003	3.0000	-0.0001
4.0000	-0.0045	4.0000	-0.0023	4.0000	-0.0011	4.0000	-0.0056	4.0000	-0.0028	4.0000	-0.0014	4.0000	-0.0056	4.0000	-0.0028	4.0000	-0.0014
5.0000	-0.0005	5.0000	-0.0003	5.0000	-0.0001	5.0000	0.0000	5.0000	0.0000	5.0000	0.0000	5.0000	0.0000	5.0000	0.0000	5.0000	0.0000
6.0000	-0.0092	6.0000	-0.0046	6.0000	-0.0023	6.0000	-0.0112	6.0000	-0.0056	6.0000	-0.0028	6.0000	-0.0093	6.0000	-0.0047	6.0000	-0.0023
7.0000	0.0005	7.0000	0.0003	7.0000	0.0001	7.0000	0.0000	7.0000	0.0000	7.0000	0.0000	7.0000	0.0000	7.0000	0.0000	7.0000	0.0000
8.0000	-0.0092	8.0000	-0.0046	8.0000	-0.0023	8.0000	-0.0112	8.0000	-0.0056	8.0000	-0.0028	8.0000	-0.0086	8.0000	-0.0043	8.0000	-0.0021
9.0000	0.0011	9.0000	0.0006	9.0000	0.0003	9.0000	0.0053	9.0000	0.0026	9.0000	0.0013	9.0000	0.0000	9.0000	0.0000	9.0000	0.0000
10.0000	-0.0045	10.0000	-0.0023	10.0000	-0.0011	10.0000	-0.0056	10.0000	-0.0028	10.0000	-0.0014	10.0000	-0.0093	10.0000	-0.0047	10.0000	-0.0023
11.0000	-0.0000	11.0000	-0.0000	11.0000	-0.0000	11.0000	0.0000	11.0000	0.0000	11.0000	0.0000	11.0000	0.0005	11.0000	0.0003	11.0000	0.0001
12.0000	0.0000	12.0000	0.0000	12.0000	0.0000	12.0000	-0.0000	12.0000	-0.0000	12.0000	-0.0000	12.0000	-0.0056	12.0000	-0.0028	12.0000	-0.0014
13.0000	-0.0000	13.0000	-0.0000	13.0000	-0.0000	13.0000	0.0000	13.0000	0.0000	13.0000	0.0000	13.0000	0.0000	13.0000	0.0000	13.0000	0.0000
14.0000	-0.0017	14.0000	-0.0008	14.0000	-0.0004	14.0000	-0.0000	14.0000	-0.0000	14.0000	-0.0000	14.0000	0.0000	14.0000	0.0000	14.0000	0.0000
15.0000	-0.0000	15.0000	-0.0000	15.0000	-0.0000	15.0000	0	15.0000	0	15.0000	0	15.0000	0.0000	15.0000	0.0000	15.0000	0.0000
16.0000	-0.0072	16.0000	-0.0036	16.0000	-0.0018	16.0000	-0.0112	16.0000	-0.0056	16.0000	-0.0028	16.0000	0.0000	16.0000	0.0000	16.0000	0.0000
stresses =		stresses =		stresses =		stresses =		stresses =		stresses =		stresses =		stresses =		stresses =	
1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *	
0.0010	7.7597	0.0010	3.8798	0.0010	1.9399	0.0010	9.6000	0.0010	4.8000	0.0010	2.4000	0.0010	9.6000	0.0010	4.8000	0.0010	2.4000
0.0020	-2.6830	0.0020	-1.3415	0.0020	-0.6707	0.0020	0.0000	0.0020	0.0000	0.0020	0.0000	0.0020	1.6973	0.0020	0.8487	0.0020	0.4243
0.0030	3.6564	0.0030	1.8282	0.0030	0.9141	0.0030	-0.0000	0.0030	-0.0000	0.0030	-0.0000	0.0030	-3.6223	0.0030	-1.8111	0.0030	-0.9056
0.0040	-2.3419	0.0040	-1.1709	0.0040	-0.5855	0.0040	9.6000	0.0040	4.8000	0.0040	2.4000	0.0040	6.4000	0.0040	3.2000	0.0040	1.6000
0.0050	7.9936	0.0050	3.9968	0.0050	1.9984	0.0050	0	0.0050	0	0.0050	0	0.0050	-0.0000	0.0050	-0.0000	0.0050	-0.0000
0.0060	-2.3419	0.0060	-1.1709	0.0060	-0.5855	0.0060	0.0000	0.0060	0.0000	0.0060	0.0000	0.0060	-3.6223	0.0060	-1.8111	0.0060	-0.9056
0.0070	1.7041	0.0070	0.8521	0.0070	0.4260	0.0070	-0.0000	0.0070	-0.0000	0.0070	-0.0000	0.0070	-0.0000	0.0070	-0.0000	0.0070	-0.0000
0.0080	-2.3419	0.0080	-1.1709	0.0080	-0.5855	0.0080	9.6000	0.0080	4.8000	0.0080	2.4000	0.0080	6.4000	0.0080	3.2000	0.0080	1.6000
0.0090	7.9936	0.0090	3.9968	0.0090	1.9984	0.0090	0.0000	0.0090	0.0000	0.0090	0.0000	0.0090	1.6973	0.0090	0.8487	0.0090	0.4243
0.0100	-2.3419	0.0100	-1.1709	0.0100	-0.5855	0.0100	-0.0000	0.0100	-0.0000	0.0100	-0.0000	0.0100	9.6000	0.0100	4.8000	0.0100	2.4000
0.0110	-2.6830	0.0110	-1.3415	0.0110	-0.6707	0.0110	9.6000	0.0110	4.8000	0.0110	2.4000	0.0110	-0.0000	0.0110	-0.0000	0.0110	-0.0000
0.0120	7.7597	0.0120	3.8798	0.0120	1.9399	0.0120	0.0000	0.0120	0.0000	0.0120	0.0000	0.0120	0.0000	0.0120	0.0000	0.0120	0.0000
0.0130	-2.6830	0.0130	-1.3415	0.0130	-0.6707	0.0130	-0.0000	0.0130	-0.0000	0.0130	-0.0000						
0.0140	-2.6830	0.0140	-1.3415	0.0140	-0.6707												

D 0.25"		D 0.5"		D 1"			C 0.25"		C 0.5"		C 1"		B 0.25"		B 0.5"		B 1"	
displ =		displ =		displ =			displ =		displ =		displ =		displ =		displ =		displ =	
1.0000	-0.0000	1.0000	-0.0000	1.0000	-0.0000		1.0000	-0.0000	1.0000	-0.0000	1.0000	-0.0000	1.0000	0	1.0000	0	1.0000	0
2.0000	0.0000	2.0000	0.0000	2.0000	0.0000		2.0000	-0.0000	2.0000	-0.0000	2.0000	-0.0000	2.0000	-0.0000	2.0000	-0.0000	2.0000	-0.0000
3.0000	0.0000	3.0000	0.0000	3.0000	0.0000		3.0000	0.0053	3.0000	0.0026	3.0000	0.0013	3.0000	-0.0053	3.0000	-0.0026	3.0000	-0.0013
4.0000	-0.0034	4.0000	-0.0017	4.0000	-0.0008		4.0000	-0.0056	4.0000	-0.0028	4.0000	-0.0014	4.0000	-0.0056	4.0000	-0.0028	4.0000	-0.0014
5.0000	-0.0006	5.0000	-0.0003	5.0000	-0.0002		5.0000	-0.0000	5.0000	-0.0000	5.0000	-0.0000	5.0000	-0.0105	5.0000	-0.0053	5.0000	-0.0026
6.0000	-0.0068	6.0000	-0.0034	6.0000	-0.0017		6.0000	-0.0112	6.0000	-0.0056	6.0000	-0.0028	6.0000	-0.0112	6.0000	-0.0056	6.0000	-0.0028
7.0000	0.0006	7.0000	0.0003	7.0000	0.0002		7.0000	-0.0000	7.0000	-0.0000	7.0000	-0.0000	7.0000	-0.0105	7.0000	-0.0053	7.0000	-0.0026
8.0000	-0.0068	8.0000	-0.0034	8.0000	-0.0017		8.0000	-0.0112	8.0000	-0.0056	8.0000	-0.0028	8.0000	-0.0112	8.0000	-0.0056	8.0000	-0.0028
9.0000	0.0000	9.0000	0.0000	9.0000	0.0000		9.0000	0.0053	9.0000	0.0026	9.0000	0.0013	9.0000	-0.0053	9.0000	-0.0026	9.0000	-0.0013
10.0000	-0.0034	10.0000	-0.0017	10.0000	-0.0008		10.0000	-0.0056	10.0000	-0.0028	10.0000	-0.0014	10.0000	-0.0056	10.0000	-0.0028	10.0000	-0.0014
11.0000	0.0000	11.0000	0.0000	11.0000	0.0000		11.0000	0	11.0000	0	11.0000	0	11.0000	0	11.0000	0	11.0000	0
12.0000	0	12.0000	0	12.0000	0		12.0000	0	12.0000	0	12.0000	0	12.0000	0	12.0000	0	12.0000	0
13.0000	0.0000	13.0000	0.0000	13.0000	0.0000													
14.0000	-0.0032	14.0000	-0.0016	14.0000	-0.0008													
							stresses =		stresses =		stresses =		stresses =		stresses =		stresses =	
stresses =		stresses =		stresses =			1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *	
1.0e+003 *		1.0e+003 *		1.0e+003 *			0.0010	0.0000	0.0010	0.0000	0.0010	0.0000	0.0010	9.6000	0.0010	4.8000	0.0010	2.4000
							0.0020	9.6000	0.0020	4.8000	0.0020	2.4000	0.0020	0.0000	0.0020	0.0000	0.0020	0.0000
0.0010	5.7981	0.0010	2.8990	0.0010	1.4495		0.0030	0	0.0030	0	0.0030	0	0.0030	-0.0000	0.0030	-0.0000	0.0030	-0.0000
0.0020	0	0.0020	0	0.0020	0		0.0040	9.6000	0.0040	4.8000	0.0040	2.4000	0.0040	9.6000	0.0040	4.8000	0.0040	2.4000
0.0030	5.7981	0.0030	2.8990	0.0030	1.4495		0.0050	-0.0000	0.0050	-0.0000	0.0050	-0.0000	0.0050	0.0000	0.0050	0.0000	0.0050	0.0000
0.0040	-4.3036	0.0040	-2.1518	0.0040	-1.0759		0.0060	-0.0000	0.0060	-0.0000	0.0060	-0.0000	0.0060	0	0.0060	0	0.0060	0
0.0050	2.0166	0.0050	1.0083	0.0050	0.5041		0.0070	9.6000	0.0070	4.8000	0.0070	2.4000	0.0070	9.6000	0.0070	4.8000	0.0070	2.4000
0.0060	-4.3036	0.0060	-2.1518	0.0060	-1.0759		0.0080	9.6000	0.0080	4.8000	0.0080	2.4000	0.0080	9.6000	0.0080	4.8000	0.0080	2.4000
0.0070	5.7981	0.0070	2.8990	0.0070	1.4495													
0.0080	0	0.0080	0	0.0080	0													
0.0090	5.7981	0.0090	2.8990	0.0090	1.4495													
0.0100	-4.3036	0.0100	-2.1518	0.0100	-1.0759													
0.0110	-4.3036	0.0110	-2.1518	0.0110	-1.0759													

F	0.25"	F	0.5"	F	1"		A	0.25"	A	0.5"	A	1"
displ =		displ =		displ =			displ =		displ =		displ =	
1.0000	-0.0000	1.0000	-0.0000	1.0000	-0.0000		1.0000	0	1.0000	0	1.0000	0
2.0000	0.0000	2.0000	0.0000	2.0000	0.0000		2.0000	0.0000	2.0000	0.0000	2.0000	0.0000
3.0000	-0.0006	3.0000	-0.0003	3.0000	-0.0002		3.0000	-0.0211	3.0000	-0.0105	3.0000	-0.0053
4.0000	-0.0068	4.0000	-0.0034	4.0000	-0.0017		4.0000	-0.0112	4.0000	-0.0056	4.0000	-0.0028
5.0000	0.0006	5.0000	0.0003	5.0000	0.0002		5.0000	-0.0211	5.0000	-0.0105	5.0000	-0.0053
6.0000	-0.0068	6.0000	-0.0034	6.0000	-0.0017		6.0000	-0.0112	6.0000	-0.0056	6.0000	-0.0028
7.0000	0	7.0000	0	7.0000	0		7.0000	0	7.0000	0	7.0000	0
8.0000	0	8.0000	0	8.0000	0		8.0000	0	8.0000	0	8.0000	0
9.0000	0.0000	9.0000	0.0000	9.0000	0.0000							
10.0000	-0.0032	10.0000	-0.0016	10.0000	-0.0008							
							stresses =		stresses =		stresses =	
stresses =		stresses =		stresses =			1.0e+003 *		1.0e+003 *		1.0e+003 *	
1.0e+003 *		1.0e+003 *		1.0e+003 *			0.0010	9.6000	0.0010	4.8000	0.0010	2.4000
							0.0020	0.0000	0.0020	0.0000	0.0020	0.0000
0.0010	5.7981	0.0010	2.8990	0.0010	1.4495		0.0030	0.0000	0.0030	0.0000	0.0030	0.0000
0.0020	-4.3036	0.0020	-2.1518	0.0020	-1.0759		0.0040	9.6000	0.0040	4.8000	0.0040	2.4000
0.0030	2.0166	0.0030	1.0083	0.0030	0.5041							
0.0040	4.3036	0.0040	2.1518	0.0040	1.0759							
0.0050	5.7981	0.0050	2.8990	0.0050	1.4495							
0.0060	-4.3036	0.0060	-2.1518	0.0060	-1.0759							
0.0070	-4.3036	0.0070	-2.1518	0.0070	-1.0759							

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APPENDIX C

Material properties from [19]

Material	Cost ($/kg)	Relative Cost	ρ (g/cm^3) [lbm/in^3]	E (Gpa) [10^6 psi]	Poisson's ratio (ν)	σY (Mpa) [ksi]	σt (Mpa) [ksi]	% elongation
Steel Alloy A36, plate, hot rolled	0.50-0.90	1	7.85 [0.283]	207 [30]	0.3	220-250 [32-36]	400-500 [58-72.5]	23
Stainless steel Alloy 304, plate	2.15-3.50	4	8.00 [0.289]	193 [28]	0.3	205 [30]	515 [75]	40
Cast Iron G1800	3.30	4.7	7.30 [0.264]	66-97 [9.6-14]	0.26		124 [18]	
Cast Iron Grade 60-40-18	3.30-5.00	5.9	7.10 [0.256]	169 [24.5]	0.29	276 [40]	414 [60]	18
Aluminum Alloy 7075, sheet T6	9.00-9.70	13.4	2.80 [0.101]	71 [10.3]	0.33	393 [57]	468 [68]	11
Aluminum Alloy 356, custom cast T6	11.00	15.7	2.69 [0.0971]	72.4 [10.5]	0.33	164 [24]	228 [33]	3.5
Copper C17200, sheet with 330C age	25-47.00	51.4	8.25 [0.298]	128 [18.6]	0.3	965-1205 [140-175]	1140-1310 [165-190]	4-10
Magnesium Alloy AZ31B, sheet	11.00	15.7	1.77 [0.0639]	45 [6.5]	0.35	220 [32]	290 [42]	15
Titanium Alloy, Commercial purity ASTM grade 1	28-65.00	66.4	4.51 [0.163]	103 [14.9]	0.34	170 [25]	240 [35]	30
Gold (cold worked)	9,500-10,250	14100	19.32 [0.697]	77 [11.2]	0.42	205 [30]	220 [32]	4
Platinum (cold worked)	11,400-14,400	18400	21.45 [0.774]	171 [24.8]	0.39		205-240 [30-35]	1-3
Silver (cold worked)	170-210.00	271	10.49 [0.379]	74 [10.7]	0.37		296 [43]	3.5
Nickel 200	19-25.00	31.4	8.89 [0.321]	204 [29.6]	0.31	148 [21.5]	462 [67]	47
Monel 400	15.50-16.50	22.9	8.80 [0.318]	180 [26]	0.32	240 [35]	550 [80]	40
Tin, Commercial purity (99.9+%)	6.85-8.85	11.2	7.17 [0.259]	44.3 [6.4]	0.33	11 [1.6]		57
Zinc, Commercial purity, ingot	1.20	1.7	7.14 [0.258]	104.5 [15.2]	0.25		134-159 [19.4-23]	50-65

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APPENDIX D

M_Elasticity [E] (E^6 psi)	Density (lbs/in^3)	Truss Width (in)	Mass of Full Wall (in^3)	A (in^2)	Volume of Voids (in^3)	VPR (%)	Mass One Wall (lbs)	s max (psi)	d max (in)	F=s*A (lbsf)	k=F/d (lbs/in)
32.25	0.38045	0.25	37.88679	0.0625	17.40453	0.911471	26.61	5798.1	0.0022	362.3813	164718.8
32.25	0.38045	0.625	37.88679	0.15625	14.98602	0.784814	28.18	2319.2	0.000863	362.375	419901.5
47.56104161	0.568671756	0.847976	56.6307	0.211994	13.61462	0.712994	43.45	1709.4	0.000431	362.3827	840210.3
16.93895839	0.192228244	0.402024	19.14289	0.100506	16.40712	0.859237	13.77	3605.6	0.0026	362.3841	139378.5
47.56104161	0.568671756	0.402024	56.6307	0.100506	16.40712	0.859237	40.75	3605.6	0.00091	362.3841	398355.6
32.25	0.38045	0.625	37.88679	0.15625	14.98602	0.784814	28.18	2319.2	0.000863	362.375	419901.5
16.93895839	0.192228244	0.847976	19.14289	0.211994	13.61462	0.712994	14.69	1709.4	0.0012	362.3827	301985.6
32.25	0.38045	1	37.88679	0.25	12.70813	0.665521	29.66	1449.5	0.000539	362.375	671811.3
32.25	0.697	0.625	69.41016	0.15625	14.98602	0.784814	51.63	2319.2	0.000863	362.375	419901.5
47.56104161	0.192228244	0.847976	19.14289	0.211994	13.61462	0.712994	14.69	1709.4	0.000431	362.3827	840210.3
16.93895839	0.568671756	0.847976	56.6307	0.211994	13.61462	0.712994	43.45	1709.4	0.0012	362.3827	301985.6
32.25	0.38045	0.625	37.88679	0.15625	14.98602	0.784814	28.18	2319.2	0.000863	362.375	419901.5
16.93895839	0.568671756	0.402024	56.6307	0.100506	16.40712	0.859237	40.75	3605.6	0.0026	362.3841	139378.5
32.25	0.0639	0.625	6.363428	0.15625	14.98602	0.784814	4.73	2319.2	0.000863	362.375	419901.5
6.5	0.38045	0.625	37.88679	0.15625	14.98602	0.784814	28.18	2319.2	0.0043	362.375	84273.26
32.25	0.38045	0.625	37.88679	0.15625	14.98602	0.784814	28.18	2319.2	0.000863	362.375	419901.5
47.56104161	0.192228244	0.402024	19.14289	0.100506	16.40712	0.859237	13.77	3605.6	0.00091	362.3841	398355.6
32.25	0.38045	0.625	37.88679	0.15625	14.98602	0.784814	28.18	2319.2	0.000863	362.375	419901.5
32.25	0.38045	0.625	37.88679	0.15625	14.98602	0.784814	28.18	2319.2	0.000863	362.375	419901.5
58	0.38045	0.625	37.88679	0.15625	14.98602	0.784814	28.18	2319.2	0.00048	362.375	755262.6

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APPENDIX E

FEMF1a

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=5;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
```

```
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);

ff(3)=600;
ff(4)=-600;
ff(5)=600;
ff(6)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
```

```
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF1b

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=5;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(3)=-600;
ff(4)=-600;
ff(5)=-600;
ff(6)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end
```

```
    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF1c

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=5;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(4)=600;
ff(6)=600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
```

```
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF1d

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=5;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(3)=600;
ff(4)=600;
ff(5)=600;
ff(6)=600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end
```

```
    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF1e

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=3;nodes(5,2)=4;
nodes(6,1)=4;nodes(6,2)=5;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=1;
bcval(1)=0;
bcdof(2)=2;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(3)=-600;
ff(4)=600;
ff(5)=-600;
ff(6)=600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end
```

```
    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF1f

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=4;nodes(5,2)=5;
nodes(6,1)=4;nodes(6,2)=1;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=5;
bcval(1)=0;
bcdof(2)=6;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(2)=-600;
ff(4)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
```

```
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF1g

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=4;nodes(5,2)=5;
nodes(6,1)=4;nodes(6,2)=1;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=5;
bcval(1)=0;
bcdof(2)=6;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(1)=600;
ff(2)=-600;
ff(3)=600;
ff(4)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end
```

```
    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF1h

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=4;nodes(5,2)=5;
nodes(6,1)=4;nodes(6,2)=1;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=5;
bcval(1)=0;
bcdof(2)=6;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(1)=-600;
ff(2)=-600;
ff(3)=-600;
ff(4)=-600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end
```

```
    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF1i

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=4;nodes(5,2)=5;
nodes(6,1)=4;nodes(6,2)=1;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=5;
bcval(1)=0;
bcdof(2)=6;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(2)=600;
ff(4)=600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
```

```
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF1j

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=4;nodes(5,2)=5;
nodes(6,1)=4;nodes(6,2)=1;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=5;
bcval(1)=0;
bcdof(2)=6;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(1)=600;
ff(2)=600;
ff(3)=600;
ff(4)=600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end
```

```
    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

FEMF1k

```
nel=7;
nnel=2;
ndof=2;
nnode=5;
sdof=nnode*ndof;

gcoord(1,1)=0;gcoord(1,2)=0;
gcoord(2,1)=0;gcoord(2,2)=12;
gcoord(3,1)=6.365;gcoord(3,2)=12;
gcoord(4,1)=6.365;gcoord(4,2)=0;
gcoord(5,1)=3.1825;gcoord(5,2)=6;

elprop(1,1)=10300000;
elprop(1,2)=.25;
elprop(2,1)=10300000;
elprop(2,2)=.25;
elprop(3,1)=10300000;
elprop(3,2)=.25;
elprop(4,1)=10300000;
elprop(4,2)=.25;
elprop(5,1)=10300000;
elprop(5,2)=.25;
elprop(6,1)=10300000;
elprop(6,2)=.25;
elprop(7,1)=10300000;
elprop(7,2)=.25;

nodes(1,1)=1;nodes(1,2)=2;
nodes(2,1)=2;nodes(2,2)=5;
nodes(3,1)=2;nodes(3,2)=3;
nodes(4,1)=3;nodes(4,2)=5;
nodes(5,1)=4;nodes(5,2)=5;
nodes(6,1)=4;nodes(6,2)=1;
nodes(7,1)=5;nodes(7,2)=1;

bcdof(1)=5;
bcval(1)=0;
bcdof(2)=6;
bcval(2)=0;
bcdof(3)=7;
bcval(3)=0;
bcdof(4)=8;
bcval(4)=0;

ff=zeros(sdof,1);
kk=zeros(sdof,sdof);
index=zeros(nnel*ndof,1);
elforce=zeros(nnel*ndof,1);
eldisp=zeros(nnel*ndof,nnel*ndof);
stress=zeros(nel,1);
```

```
ff(1)=-600;
ff(2)=600;
ff(3)=-600;
ff(4)=600;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end

    el=elprop(iel,1);
    area=elprop(iel,2);
    [index]=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    kk=FEASMBL1(kk,k,index);

end

[kk,ff]=FEAPLYC2(kk,ff,bcdof,bcval);

disp=kk\ff;

for iel=1:nel

    nd(1)=nodes(iel,1);
    nd(2)=nodes(iel,2);

    x1=gcoord(nd(1),1);y1=gcoord(nd(1),2);
    x2=gcoord(nd(2),1);y2=gcoord(nd(2),2);

    leng=sqrt((x2-x1)^2+(y2-y1)^2);

    if (x2-x1)==0;
        beta=2*atan(1);
    else
        beta=atan((y2-y1)/(x2-x1));
    end
```

```
    el=elprop(iel,1);
    area=elprop(iel,2);
    index=FEELDOF(nd,nnel,ndof);

    k=FETRUSS2(el,leng,area,0,beta,1);

    for i=1:(nnel*ndof)
        eldisp(i)=disp(index(i));
    end

    elforce=k*eldisp;
    stress(iel)=sqrt(elforce(1)^2+elforce(2)^2)/area;

    if((x2-x1)*elforce(3))<0;
        stress(iel)=-stress(iel);
    end

end

num=1:1:sdof;
displ=[num' disp]

numm=1:1:nel;
stresses=[numm' stress]
```

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APPENDIX F

Stress and deflections from compound loading of Configuration F

F1		F1a		F1b		F1c		F1d		F1e		F1f		F1g	
displ =		displ =		displ =		displ =		displ =		displ =		displ =		displ =	
1.0000	-0.0000	1.0000	0.0000	1.0000	-0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	-0.0000	1.0000	0.0008	1.0000	0.0022
2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	-0.0000	2.0000	-0.0000	2.0000	-0.0000	2.0000	-0.0045	2.0000	-0.0047
3.0000	-0.0002	3.0000	0.0242	3.0000	-0.0245	3.0000	0.0002	3.0000	0.0245	3.0000	-0.0242	3.0000	-0.0008	3.0000	0.0006
4.0000	-0.0017	4.0000	0.0036	4.0000	-0.0070	4.0000	0.0017	4.0000	0.0070	4.0000	-0.0036	4.0000	-0.0045	4.0000	-0.0043
5.0000	0.0002	5.0000	0.0245	5.0000	-0.0242	5.0000	-0.0002	5.0000	0.0242	5.0000	-0.0245	5.0000	0.0000	5.0000	-0.0000
6.0000	-0.0017	6.0000	-0.0070	6.0000	0.0036	6.0000	0.0017	6.0000	-0.0036	6.0000	0.0070	6.0000	-0.0000	6.0000	0.0000
7.0000	0	7.0000	-0.0000	7.0000	0.0000	7.0000	0	7.0000	-0.0000	7.0000	0.0000	7.0000	0.0000	7.0000	-0.0000
8.0000	0	8.0000	0.0000	8.0000	0.0000	8.0000	0	8.0000	-0.0000	8.0000	-0.0000	8.0000	0.0000	8.0000	-0.0000
9.0000	0.0000	9.0000	0.0072	9.0000	-0.0072	9.0000	-0.0000	9.0000	0.0072	9.0000	-0.0072	9.0000	-0.0000	9.0000	0.0005
10.0000	-0.0008	10.0000	-0.0008	10.0000	-0.0008	10.0000	0.0008	10.0000	0.0008	10.0000	0.0008	10.0000	-0.0020	10.0000	-0.0020
stresses =		stresses =		stresses =		stresses =		stresses =		stresses =		stresses =		stresses =	
1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *	
0.0010	1.4495	0.0010	3.0752	0.0010	5.9743	0.0010	1.4495	0.0010	5.9743	0.0010	3.0752	0.0010	0	0.0010	0.3216
0.0020	-1.0759	0.0020	-6.1978	0.0020	4.0459	0.0020	1.0759	0.0020	-4.0459	0.0020	6.1978	0.0020	-2.7167	0.0020	-3.0807
0.0030	0.5041	0.0030	0.5041	0.0030	0.5041	0.0030	-0.5041	0.0030	-0.5041	0.0030	-0.5041	0.0030	1.2730	0.0030	-0.9564
0.0040	-1.0759	0.0040	4.0459	0.0040	-6.1978	0.0040	1.0759	0.0040	6.1978	0.0040	-4.0459	0.0040	2.7167	0.0040	2.3527
0.0050	1.4495	0.0050	5.9743	0.0050	3.0752	0.0050	1.4495	0.0050	3.0752	0.0050	5.9743	0.0050	-2.7167	0.0050	-3.0807
0.0060	-1.0759	0.0060	-6.1978	0.0060	4.0459	0.0060	1.0759	0.0060	-4.0459	0.0060	6.1978	0.0060	-1.2730	0.0060	-3.5024
0.0070	-1.0759	0.0070	4.0459	0.0070	-6.1978	0.0070	1.0759	0.0070	6.1978	0.0070	-4.0459	0.0070	2.7167	0.0070	2.3527

F1h		F1i		F1j		F1k	
displ =		displ =		displ =		displ =	
1.0000	-0.0006	1.0000	-0.0008	1.0000	0.0006	1.0000	-0.0022
2.0000	-0.0043	2.0000	0.0045	2.0000	0.0043	2.0000	0.0047
3.0000	-0.0022	3.0000	0.0008	3.0000	0.0022	3.0000	-0.0006
4.0000	-0.0047	4.0000	0.0045	4.0000	0.0047	4.0000	0.0043
5.0000	0.0000	5.0000	-0.0000	5.0000	-0.0000	5.0000	0.0000
6.0000	-0.0000	6.0000	0.0000	6.0000	0.0000	6.0000	-0.0000
7.0000	-0.0000	7.0000	-0.0000	7.0000	0.0000	7.0000	0.0000
8.0000	-0.0000	8.0000	-0.0000	8.0000	0.0000	8.0000	0.0000
9.0000	-0.0005	9.0000	0.0000	9.0000	0.0005	9.0000	-0.0005
10.0000	-0.0020	10.0000	0.0020	10.0000	0.0020	10.0000	0.0020
stresses =		stresses =		stresses =		stresses =	
1.0e+003 *		1.0e+003 *		1.0e+003 *		1.0e+003 *	
0.0010	0.3216	0.0010	0	0.0010	0.3216	0.0010	0.3216
0.0020	-2.3527	0.0020	2.7167	0.0020	2.3527	0.0020	3.0807
0.0030	3.5024	0.0030	-1.2730	0.0030	-3.5024	0.0030	0.9564
0.0040	3.0807	0.0040	-2.7167	0.0040	-3.0807	0.0040	-2.3527
0.0050	-2.3527	0.0050	2.7167	0.0050	2.3527	0.0050	3.0807
0.0060	0.9564	0.0060	1.2730	0.0060	-0.9564	0.0060	3.5024
0.0070	3.0807	0.0070	-2.7167	0.0070	-3.0807	0.0070	-2.3527

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APPENDIX G

	Absolute Max deflection	Absolute Max Stress	σ (psi)	A (in^2)	F=σ*A (lbs)	k=F/δ (lbs/in)	% of full k	P_{cr} (lbs)	Status
F1	0.0017	1.4495	1449.5	0.25	362.375	213,162	15.60	919.2069	Safe
F1a1	0.0245	6.1978	6197.8	0.25	1549.45	63,243	4.63	919.2069	Failure
F1b1	0.0245	6.1978	6197.8	0.25	1549.45	63,243	4.63	919.2069	Failure
F1c1	0.0017	1.4495	1449.5	0.25	362.375	213,162	15.60	919.2069	Safe
F1d1	0.0245	6.1978	6197.8	0.25	1549.45	63,243	4.63	919.2069	Failure
F1e1	0.0245	6.1978	6197.8	0.25	1549.45	63,243	4.63	919.2069	Failure
F1f1	0.0045	2.7167	2716.7	0.25	679.175	150,928	11.05	919.2069	Safe
F1g1	0.0047	3.5024	3502.4	0.25	875.6	186,298	13.64	919.2069	Safe
F1h1	0.0047	3.5024	3502.4	0.25	875.6	186,298	13.64	919.2069	Safe
F1i1	0.0045	2.7167	2716.7	0.25	679.175	150,928	11.05	919.2069	Safe
F1j1	0.0047	3.5024	3502.4	0.25	875.6	186,298	13.64	919.2069	Safe
F1k1	0.0047	3.5024	3502.4	0.25	875.6	186,298	13.64	919.2069	Safe

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